

# Package ‘archetypal’

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**Version** 1.3.0

**Title** Finds the Archetypal Analysis of a Data Frame

**Description** Performs archetypal analysis by using Principal Convex Hull Analysis under a full control of all algorithmic parameters.

It contains a set of functions for determining the initial solution, the optimal algorithmic parameters and the optimal number of archetypes.

Post run tools are also available for the assessment of the derived solution.

Morup, M., Hansen, LK (2012) <doi:10.1016/j.neucom.2011.06.033>.

Hochbaum, DS, Shmoys, DB (1985) <doi:10.1287/moor.10.2.180>.

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archetypal-package	<i>Finds the Archetypal Analysis of a Data Frame</i>
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**Description**

Performs archetypal analysis by using Principal Convex Hull Analysis (PCHA) under a full control of all algorithmic parameters. It contains a set of functions for determining the initial solution, the optimal algorithmic parameters and the optimal number of archetypes. Post run tools are also available for the assessment of the derived solution.

### Compute Archetypal Analysis (AA)

The main function is `archetypal` which is a variant of PCHA algorithm, see [1], [2], suitable for R language. It provides control to the entire set of involved parameters and has two main options:

1. `initialrows = NULL`, then a method from "projected\_convexhull", "convexhull", "partitioned\_convexhul", "furthestsum", "outmost", "random" is used
2. `initialrows = (a vector of kappas rows)`, then given rows form the initial solution for AA

This is the main function of the package, but extensive trials has shown that:

- AA may be very difficult to run if a random initial solution has been chosen
- for the same data set the final Sum of Squared Errors (SSE) may be much smaller if initial solution is close to the final one
- even the quality of AA done is affected from the starting point

This is the reason why we have developed a whole set of methods for choosing initial solution for the PCHA algorithm.

### Find a time efficient initial approximation for AA

There are three functions that work with the Convex Hull (CH) of data set.

1. `find_outmost_convexhull_points` computes the CH of all points
2. `find_outmost_projected_convexhull_points` computes the CH for all possible combinations of variables taken by `npr` (default=2)
3. `find_outmost_partitioned_convexhull_points` makes `np` partitions of data frame (default=10), then computes CH for each partition and finally gives the CH of overall union

The most simple method for estimating an initial solution is `find_outmost_points` where we just compute the outermost points, i.e. those that are the most frequent outermost for all available points.

The default method "FurthestSum" (FS) of PCHA (see [1], [2]) is used by `find_furthestsum_points` which applies FS for `nfurthest` times (default=10) and then finds the most frequent points.

Of course "random" method is available for comparison reasons and that gives a random set of kappas points as initial solution.

All methods give the number of rows for the input data frame as integers. Attention needed if your data frame has row names which are integers but not identical to `1:dim(df)[1]`.

### Find the optimal number of archetypes

For that task `find_optimal_kappas` is available which runs for each kappas from 1 to `maxkappas` (default=15) `ntrials` (default=10) times AA, stores SSE, VarianceExplained from each run and then computes knee or elbow point by using UIK method, see [3].

### Determining the optimal updating parameters

Extensive trials have shown us that choosing the proper values for algorithmic updating parameters (`muAup`, `muAdown`, `muBup`, `muBdown`) can speed up remarkably the process. That is the task of `find_pcha_optimal_parameters` which conducts a grid search with different values of these parameters and returns the values which minimize the SSE after a fixed number of iterations (`testing_iters`, default=10).

### Evaluate the quality of Archetypal Analysis

By using function `check_Bmatrix` we can evaluate the overall quality of applied method and algorithm. Quality can be considered high:

1. if every archetype is being created by a small number of data points
2. if relevant weights are not numerically insignificant

Of course we must take into account the SSE and VarianceExplained, but if we have to compare two solutions with similar termination status, then we must choose that of the simplest B matrix form.

### Resampling

The package includes a function for resampling (`grouped_resample`) which may be used for standard bootstrapping or for subsampling. This function allows samples to be drawn with or without replacement, by groups and with or without Dirichlet weights. This provides a variety of options for researchers who wish to correct sample biases, estimate empirical confidence intervals, and/or subsample large data sets.

### Post-run tools

Except from `check_Bmatrix` there exist next functions for checking the convergence process itself and for examining the local neighborhood of archetypes:

1. The function `study_AAconvergence` analyzes the history of iterations done and produces a multi-panel plot showing the steps and quality of the convergence to the final archetypes.
2. By setting the desired number `npoints` as argument in function `find_closer_points` we can then find the data points that are in the local neighborhood of each archetype. This allows us to study the properties of the solution or manually choose an initial approximation to search for a better fit.

### Note

Bug reports and feature requests can be sent to <dchristop@econ.uoa.gr> or <dem.christop@gmail.com>.

### Author(s)

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## References

- [1] M Morup and LK Hansen, "Archetypal analysis for machine learning and data mining", *Neuro-computing* (Elsevier, 2012). <https://doi.org/10.1016/j.neucom.2011.06.033>.
- [2] Source: [https://www.mortenmorup.dk/MMhomepageUpdated\\_files/Page327.htm](https://www.mortenmorup.dk/MMhomepageUpdated_files/Page327.htm) , last accessed 2021-09-23
- [3] Christopoulos, Demetris T., Introducing Unit Invariant Knee (UIK) As an Objective Choice for Elbow Point in Multivariate Data Analysis Techniques (March 1, 2016). Available at SSRN: <https://ssrn.com/abstract=3043076> or <http://dx.doi.org/10.2139/ssrn.3043076>

## See Also

[archetypal](#)

---

AbsoluteTemperature	<i>Global Absolute Temperature data set for Northern Hemisphere 1969-2013</i>
---------------------	---

---

## Description

It is a subset from the data set which was used for publication [1], i.e. the Global Absolute Temperature for Northern Hemisphere (1800-2013) with only complete yearly observations included. Here we have kept the years 1969-2013.

## Usage

```
data("AbsoluteTemperature")
```

## Format

A data frame with 155862 observations on the following 18 variables.

Year an integer vector of observation years from 1969 to 2013

Jan numeric vector of monthly average temperature for January

Feb numeric vector of monthly average temperature for February

Mar numeric vector of monthly average temperature for March

Apr numeric vector of monthly average temperature for April

May numeric vector of monthly average temperature for May

Jun numeric vector of monthly average temperature for June

Jul numeric vector of monthly average temperature for July

Aug numeric vector of monthly average temperature for August

Sep numeric vector of monthly average temperature for September

Oct numeric vector of monthly average temperature for October

Nov numeric vector of monthly average temperature for November

Dec numeric vector of monthly average temperature for December  
 long a numeric vector for the geographical longitude: positive values for eastings  
 lat a numeric vector for the geographical latitude: positive values for northings  
 h a numeric vector for the altitude in metres  
 stid an integer vector with the station identity number  
 z an integer vector with the relevant climate zone:

- 1, Tropical Zone
- 2, Subtropics
- 3, Temperate zone
- 4, Cold Zone

### Details

That data set was the output of the procedure described in [1]. Initial data set was downloaded from [2] at 2014-12-17.

### References

[1] Demetris T. Christopoulos. Extraction of the global absolute temperature for Northern Hemisphere using a set of 6190 meteorological stations from 1800 to 2013. *Journal of Atmospheric and Solar-Terrestrial Physics*, 128:70 - 83, 3 2015. doi:10.1016/j.jastp.2015.03.009

[2] Met Office Hadley Centre observations datasets, station data sets,  
[http://www.metoffice.gov.uk/hadobs/crutem4/data/station\\_files/CRUTEM.4.2.0.0.station\\_files.zip](http://www.metoffice.gov.uk/hadobs/crutem4/data/station_files/CRUTEM.4.2.0.0.station_files.zip)  
 (last visited 17.12.14)

### Examples

```
#
#####
## Load absolute temperature data set:
#####
#
data("AbsoluteTemperature")
df=AbsoluteTemperature
## Find proportions for climate zones
pcs=table(df$z)/dim(df)[1]
## Choose an approximate size of the new sample and compute resample sizes
N=1000
resamplesizes=as.integer(round(N*pcs))
sum(resamplesizes)
## Create the grouping matrix
groupmat=data.frame("Group_ID"=1:4,"Resample_Size"=resamplesizes)
groupmat
## Simple resampling:
resample_simple <- grouped_resample(in_data = df,grp_vector = "z",
grp_matrix = groupmat,replace = FALSE, option = "Simple", rseed = 20191119)
cat(dim(resample_simple),"\n")
## Dirichlet resampling:
```

```

resample_dirichlet <- grouped_resample(in_data = df,grp_vector = "z",
grp_matrix = groupmat, replace = FALSE, option = "Dirichlet", rseed = 20191119)
cat(dim(resample_dirichlet),"\n")
#
#####
## Reproduce the results of 2015 article
#####
##
data("AbsoluteTemperature")
dh=AbsoluteTemperature
## Create yearly averages for every station
dh$avg = rowMeans(df[,month.abb[1:12]])
head(dh)
## Compute mean average of every year for all Northern Hemisphere
dagg=data.frame(aggregate(avg~Year,dh,function(x){c(mean(x),sd(x))}))
## Find used stations per year
daggn=aggregate(stid ~ Year,dh,length)
head(daggn)
tail(daggn)
## Combine all in a data frame
dagyears=data.frame(dagg$Year,daggn$stid,dagg$avg[,1],dagg$avg[,2])
colnames(dagyears)=c("Year","Nv","mu","Smu")
head(dagyears)
tail(dagyears)
#
## Compare with Table 7 (Columns: Year, Nv, mu_bar, Smu_bar), page 77 of article
## Extraction of the global absolute temperature for Northern Hemisphere
## using a set of 6190 meteorological stations from 1800 to 2013
## https://doi.org/10.1016/j.jastp.2015.03.009
## and specifically the years 1969--2013

```

---

align\_archetypes\_from\_list

*Align archetypes from a list either by the most frequent found or by using a given archetype*

---

## Description

Align archetypes from a list either by the most frequent or by using a given archetype.

## Usage

```

align_archetypes_from_list(archs_list, given_arch = NULL,
varnames = NULL, ndigits = 0, parallel = FALSE,
nworkers = NULL, verbose = TRUE)

```

## Arguments

archs\_list      The list of archetypes that must be aligned

given_arch	If it is not NULL, then given_arch will be used as guide for aligning other archetypes of list. Otherwise, a heuristic for finding the most frequent archetype will be used.
varnames	The character vector of variable names that must be used. If it is NULL, then the column names of first archetype will be used.
ndigits	The number of digits that will be used for truncation.
parallel	If it set to TRUE, then parallel processing will be applied.
nworkers	The number of logical processors that will be used for parallel computing (usually it is the double of available physical cores).
verbose	If it is set to TRUE, then details are printed out

### Value

A list with members:

1. arch\_guide, the archetype used as guide for aligning others
2. phrases\_most, a table with all rounded phrases from archetypes. Frequencies are in decreasing order, so first row indicates the most frequent sequence, if exists. Otherwise we take randomly a case and proceed.
3. archs\_aa\_output, a data frame with rows all given archetypes
4. archs\_aligned, the final list of aligned archetypes

### References

This function is a modification of "align\_arc" function from package "ParetoTI", see <https://github.com/vitkl/ParetoTI> and [https://github.com/vitkl/ParetoTI/blob/master/R/align\\_arc.R](https://github.com/vitkl/ParetoTI/blob/master/R/align_arc.R)

### Examples

```
data("wd2") #2D demo
df = wd2
# Define 4 archetypes found for it
dalist = list(c(2.172991,3.200754,5.384013,2.579770,4.860343,3.085111),
             c(5.430821,3.128493,2.043495,3.146342,4.781851,2.710885),
             c(5.430752,2.043403,3.128520,3.146252,2.710979,4.781880),
             c(2.043854,5.430890,3.127183,2.710522,3.146432,4.780432))
archslist = lapply(dalist, function(x){matrix(x,ncol=2)}) #not aligned
# Run aligner
yy = align_archetypes_from_list(archs_list = archslist,
                               given_arch = archslist[[1]])
yy$arch_guide
aligned_archs = yy$archs_aligned
aligned_archs #observe that they are comparable now
```



---

archetypal	<i>archetypal: Finds the archetypal analysis of a data frame by using a variant of the PCHA algorithm</i>
------------	---

---

## Description

Performs archetypal analysis by using Principal Convex Hull Analysis (PCHA) under a full control of all algorithmic parameters.

## Usage

```
archetypal(df, kappas, initialrows = NULL,
  method = "projected_convexhull", nprojected = 2, npartition = 10,
  nfurthest = 10, maxiter = 2000, conv_crit = 1e-06,
  var_crit = 0.9999, verbose = TRUE, rseed = NULL, aupdate1 = 25,
  aupdate2 = 10, bupdate = 10, muAup = 1.2, muAdown = 0.5,
  muBup = 1.2, muBdown = 0.5, SSE_A_conv = 1e-09,
  SSE_B_conv = 1e-09, save_history = FALSE, nworkers = NULL,
  stop_varexpl = TRUE)
```

## Arguments

df	The data frame with dimensions n x d
kappas	The number of archetypes
initialrows	The initial set of rows from data frame that will be used for starting algorithm
method	The method that will be used for computing initial approximation: <ol style="list-style-type: none"> <li>1. <code>projected_convexhull</code>, see <a href="#">find_outmost_projected_convexhull_points</a></li> <li>2. <code>convexhull</code>, see <a href="#">find_outmost_convexhull_points</a></li> <li>3. <code>partitioned_convexhull</code>, see <a href="#">find_outmost_partitioned_convexhull_points</a></li> <li>4. <code>furthestsum</code>, see <a href="#">find_furthestsum_points</a></li> <li>5. <code>outmost</code>, see <a href="#">find_outmost_points</a></li> <li>6. <code>random</code>, a random set of kappas points will be used</li> </ol>
nprojected	The dimension of the projected subspace for <a href="#">find_outmost_projected_convexhull_points</a>
npartition	The number of partitions for <a href="#">find_outmost_partitioned_convexhull_points</a>
nfurthest	The number of times that <code>FurthestSum</code> algorithm will be applied by <a href="#">find_furthestsum_points</a>
maxiter	The maximum number of iterations for main algorithm application
conv_crit	The SSE convergence criterion of termination: iterate until $ldSSE/SSE < conv\_crit$
var_crit	The Variance Explained (VarExpl) convergence criterion of termination: iterate until $VarExpl < var\_crit$
verbose	If it is set to TRUE, then both initialization and iteration details are printed out
rseed	The random seed that will be used for setting initial A matrix. Useful for reproducible results.

aupdate1	The number of initial applications of Aupdate for improving the initially randomly selected A matrix
aupdate2	The number of Aupdate applications in main iteration
bupdate	The number of Bupdate applications in main iteration
muAup	The factor (>1) by which muA is multiplied when it holds $SSE \leq SSE_{old}(1+SSE\_A\_conv)$
muAdown	The factor (<1) by which muA is multiplied when it holds $SSE > SSE_{old}(1+SSE\_A\_conv)$
muBup	The factor (>1) by which muB is multiplied when it holds $SSE \leq SSE_{old}(1+SSE\_B\_conv)$
muBdown	The factor (<1) by which muB is multiplied when it holds $SSE > SSE_{old}(1+SSE\_B\_conv)$
SSE_A_conv	The convergence value used in $SSE \leq SSE_{old}(1+SSE\_A\_conv)$ . Warning: there exists a Matlab crash sometimes after setting this to 1E-16 or lower
SSE_B_conv	The convergence value used in $SSE \leq SSE_{old}(1+SSE\_A\_conv)$ . Warning: there exists a Matlab crash sometimes after setting this to 1E-16 or lower
save_history	If set TRUE, then iteration history is being saved for further use
nworkers	The number of logical processors that will be used for parallel computing (usually it is the double of available physical cores). Parallel computation is applied when asked by functions <a href="#">find_furthestsum_points</a> , <a href="#">find_outmost_partitioned_convexhull_points</a> and <a href="#">find_outmost_projected_convexhull_points</a> .
stop_varexpl	If set TRUE, then algorithm stops if varexpl is greater than var_crit

## Value

A list with members:

1. BY, the  $kappas \times d$  matrix of archetypes found
2. A, the  $n \times kappas$  matrix such that  $Y \sim ABY$  or Frobenius norm  $\|Y-ABY\|$  is minimum
3. B, the  $kappas \times n$  matrix such that  $Y \sim ABY$  or Frobenius norm  $\|Y-ABY\|$  is minimum
4. SSE, the sum of squared error  $SSE = \|Y-ABY\|^2$
5. varexpl, the Variance Explained =  $(SST-SSE)/SST$  where SST is the total sum of squares for data set matrix
6. initialsolution, the initially used set of rows from data frame in order to start the algorithm
7. freqstable, the frequency table for all found rows, if it is available.
8. iterations, the number of main iterations done by algorithm
9. time, the time in seconds that was spent from entire run
10. converges, if it is TRUE, then convergence was achieved before the end of maximum allowed iterations
11. nAup, the total number of times when it was  $SSE \leq SSE_{old}(1+SSE\_A\_conv)$  in Aupdate processes. Useful for debugging purposes.
12. nAdown, the total number of times when it was  $SSE > SSE_{old}(1+SSE\_A\_conv)$  in Aupdate processes. Useful for debugging purposes.
13. nBup, the total number of times when it was  $SSE \leq SSE_{old}(1+SSE\_B\_conv)$  in Bupdate processes. Useful for debugging purposes.

14. nBdown, the total number of times when it was  $SSE > SSE_{old}(1 + SSE\_A\_conv)$  in Bupdate processes. Useful for debugging purposes.
15. run\_results, a list of iteration related details: SSE, varexpl, time, B, BY for all iterations done.
16. Y, the  $n \times d$  matrix of initial data used
17. data.tables, the initial data frame if column dimension is at most 3 or a list of frequencies for each variable
18. call, the exact calling used

## References

- [1] M Morup and LK Hansen, "Archetypal analysis for machine learning and data mining", Neuro-computing (Elsevier, 2012). <https://doi.org/10.1016/j.neucom.2011.06.033>.
- [2] Source: [http://www.mortenmorup.dk/MMhomepageUpdated\\_files/Page327.htm](http://www.mortenmorup.dk/MMhomepageUpdated_files/Page327.htm) , last accessed 2021-11-19

## Examples

```
{
# Create a small 2D data set from 3 corner-points:
p1 = c(1,2);p2 = c(3,5);p3 = c(7,3)
dp = rbind(p1,p2,p3);dp
set.seed(916070)
pts = t(sapply(1:20, function(i,dp){
  cc = runif(3)
  cc = cc/sum(cc)
  colSums(dp*cc)
},dp))
df = data.frame(pts)
colnames(df) = c("x","y")
# Run AA:
aa = archetypal(df = df, kappas = 3, verbose = FALSE, save_history = TRUE)
# Archetypes:
archs = data.frame(aa$BY)
archs
# See main results:
names(aa)
aa[c("SSE","varexpl","iterations","time")]
# See history of iterations:
names(aa$run_results)

}
```

---

check\_Bmatrix

*Function which checks B matrix of Archetypal Analysis  $Y \sim A B Y$  in order to find the used rows for creating each archetype and the relevant used weights.*

---

**Description**

Function which checks B matrix of Archetypal Analysis  $Y \sim A B Y$  in order to find the used rows for creating each archetype and the relevant used weights.

**Usage**

```
check_Bmatrix(B, chvertices = NULL, verbose = TRUE)
```

**Arguments**

B	The $kappas \times n$ matrix such that $Y \sim A B Y$ or Frobenius norm $\ Y-ABY\ $ is minimum
chvertices	The vector of rows which represent the Convex Hull of data frame
verbose	If set to TRUE, then results are printed out.

**Value**

A list with members:

1. used\_rows, a list with used rows for creating each archetype
2. used\_weights, a list with the relevant weights that have been used
3. leading\_rows, the rows for each archetype with greatest weight
4. leading\_weights, the weights of leading rows
5. used\_on\_convexhull, the portion of used rows which lie on Convex Hull (if given)

**See Also**

[archetypal](#), [check\\_Bmatrix](#), [find\\_closer\\_points](#)  
& [study\\_AAconvergence](#)

**Examples**

```
{
# Load data "wd2"
data("wd2")
df = wd2
# Run AA:
aa = archetypal(df = df, kappas = 3, verbose = FALSE)
# Check B matrix:
B = aa$B
yy = check_Bmatrix(B, verbose = TRUE)
yy$used_rows
yy$used_weights
yy$leading_rows
yy$leading_weights
# Check if used rows lie on ConvexHull
ch = chull(df)
yy = check_Bmatrix(B, chvertices = ch, verbose = FALSE)
yy$used_on_convexhull
```

```
#  
}
```

---

dirichlet\_sample      *Function which performs Dirichlet sampling*

---

### Description

It uses Dirichlet weights for creating sub-samples of initial data set.

### Usage

```
dirichlet_sample(in_data = NULL, sample_size = NULL,  
replacement = NULL, rseed = NULL)
```

### Arguments

in_data	The initial data frame that must be re-sampled. It must contain: <ol style="list-style-type: none"><li>1. an ID variable</li><li>2. the variables of interest</li><li>3. a grouping variable</li></ol>
sample_size	An integer for the size of the new sample
replacement	A logical input: TRUE/FALSE if replacement should be used or not, respectively
rseed	The random seed that will be used for setting initial A matrix. Useful for reproducible results

### Value

It returns a data frame with exactly the same variables as the initial one, except that group variable has now only the given value from input data frame.

### Author(s)

David Midgley

### See Also

[grouped\\_resample](#)

**Examples**

```
## Load absolute temperature data set:
data("AbsoluteTemperature")
df=AbsoluteTemperature
## Find portions for climate zones
pcs=table(df$z)/dim(df)[1]
## Choose the approximate size of the new sample and compute resample sizes
N=1000
resamplesizes=as.integer(round(N*pcs))
sum(resamplesizes)
## Create the grouping matrix
groupmat=data.frame("Group_ID"=1:4,"Resample_Size"=resamplesizes)
groupmat
## Dirichlet resampling:
resample_dirichlet <- grouped_resample(in_data = df,grp_vector = "z",
                                     grp_matrix = groupmat,replace = FALSE,
                                     option = "Dirichlet", rseed = 20191220)
cat(dim(resample_dirichlet),"\n")
```

---

find_closer_points	<i>Function which finds the data points that are closer to the archetypes during all iterations of the algorithm PCHA</i>
--------------------	---

---

**Description**

This function runs the PCHA algorithm and finds the data points that are in the local neighborhood of each archetype. The size of the neighborhood is user defined (npoints). This allows us to study the properties of the solution or manually choose an initial approximation to search for a better fit.

**Usage**

```
find_closer_points(df, kappas, usedata = FALSE, npoints = 2,
                  nworkers = NULL, rseed = NULL,
                  verbose = FALSE, doparallel = FALSE, ...)
```

**Arguments**

df	The data frame with dimensions n x d
kappas	The number of archetypes
usedata	If it is TRUE, then entire data frame will be used, if doparallel = TRUE
npoints	The number of closer points to be estimated
nworkers	The number of logical processors that will be used, if doparallel = TRUE
rseed	The random seed that will be used for random generator. Useful for reproducible results.
verbose	If it is set to TRUE, then details will be printed, except from archetypal
doparallel	If it is set to TRUE, then parallel processing will be performed

... Other arguments to be passed to `archetypal` except internally used `save_history = TRUE` and `verbose = FALSE`. This is essential for using optimal parameters found by `find_pcha_optimal_parameters`

### Value

A list with members:

1. `rows_history`, a list with `npoints` rows used that are closer to each archetype for each iteration done by algorithm
2. `iter_terminal`, iteration after which rows closer to archetypes do not change any more
3. `rows_closer`, the rows closer to archetypes by means of Euclidean distance and are fixed after `iter_terminal` iteration
4. `rows_closer_matrix`, a matrix with `Jcodenpoints` rows which are closer to each archetype
5. `solution_used`, the AA output that has been used. Some times useful, especially for big data.

### See Also

[check\\_Bmatrix](#), [study\\_AAconvergence](#)

### Examples

```
{
# Load data "wd2"
data("wd2")
yy = find_closer_points(df = wd2, kappas = 3, npoints = 2, nworkers = 2)
yy$rows_history
yy$iter_terminal
yy$rows_closer
yy$rows_closer_matrix
yy$solution_used$BY
}
```

---

find\_furthestsum\_points

*Function which finds the furthest sum points in order to be used as initial solution in archetypal analysis*

---

### Description

Function which finds the furthest sum points in order to be used as initial solution in archetypal analysis.

### Usage

```
find_furthestsum_points(df, kappas, nfurthest = 100, nworkers = NULL,
  sortrows = TRUE, doparallel = TRUE)
```





**Arguments**

df	The data frame with dimensions $n \times d$
maxkappas	The maximum number of archetypes for which algorithm will be applied
method	The method that will be used for computing the initial solution
ntrials	The number of times that algorithm will be applied for each kappas
nworkers	The number of logical processors that will be used for parallel computing (usually it is the double of available physical cores)
...	Other arguments to be passed to function <a href="#">archetypal</a>

**Details**

After having found the SSE for each kappas, UIK method (see [1]) is used for estimating the knee or elbow point as the optimal kappas.

**Value**

A list with members:

1. all\_sse, all available SSE for all kappas and all trials per kappas
2. all\_sse1, all available SSE(k)/SSE(1) for all kappas and all trials per kappas
3. bestfit\_sse, only the best fit SSE trial for each kappas
4. bestfit\_sse1, only the best fit SSE(k)/SSE(1) trial for each kappas
5. all\_kappas, the knee point of scree plot for all 4 SSE results
6. d2uik, the UIK for the absolute values of the estimated best fit SSE second derivatives, after using second order forward divided differences approximation
7. optimal\_kappas, the knee point from best fit SSE results

**References**

[1] Christopoulos, Demetris T., Introducing Unit Invariant Knee (UIK) As an Objective Choice for Elbow Point in Multivariate Data Analysis Techniques (March 1, 2016). Available at SSRN: <http://dx.doi.org/10.2139/ssrn.3043076>

**See Also**

[archetypal](#)

**Examples**

```
{
# Run may take a while depending on your machine ...
# Load data frame "wd2"
data("wd2")
df = wd2
# Run:
t1 = Sys.time()
yy = find_optimal_kappas(df, maxkappas = 10)
```

```

t2 = Sys.time();print(t2-t1)
# Results:
names(yy)
# Best fit SSE:
yy$bestfit_sse
# Optimal kappas from UIK method:
yy$optimal_kappas
#
}

```

---

find\_outmost\_convexhull\_points

*Function which finds the outermost convex hull points in order to be used as initial solution in archetypal analysis*

---

### Description

Function which finds the outermost convex hull points in order to be used as initial solution in archetypal analysis

### Usage

```
find_outmost_convexhull_points(df, kappas)
```

### Arguments

df	The data frame with dimensions n x d
kappas	The number of archetypes

### Details

This function uses the [chull](#) when d=2 (see [1], [2]) and the [convhulln](#) for d>2 (see [3]) cases.

### Value

A list with members:

1. outmost, the first kappas most frequent outermost points as rows of data frame
2. outmostall, all the outermost points that have been found as rows of data frame
3. outmostfrequency, a matrix with frequency and cumulative frequency for outermost rows

### References

- [1] Eddy, W. F. (1977). A new convex hull algorithm for planar sets. *ACM Transactions on Mathematical Software*, 3, 398-403. doi: 10.1145/355759.355766.
- [2] Eddy, W. F. (1977). Algorithm 523: CONVEX, A new convex hull algorithm for planar sets [Z]. *ACM Transactions on Mathematical Software*, 3, 411-412. doi: 10.1145/355759.355768.
- [3] Barber, C.B., Dobkin, D.P., and Huhdanpaa, H.T., "The Quickhull algorithm for convex hulls" *ACM Trans. on Mathematical Software*, 22(4):469-483, Dec 1996, <http://www.qhull.org>

**See Also**

[find\\_furthestsum\\_points](#), [find\\_outmost\\_projected\\_convexhull\\_points](#),  
[find\\_outmost\\_partitioned\\_convexhull\\_points](#) & [find\\_outmost\\_points](#)

**Examples**

```
data("wd2") #2D demo
df = wd2
yy = find_outmost_convexhull_points(df, kappas = 3)
yy$outmost #the rows of 3 outermost points
df[yy$outmost,] #the 3 outermost points
yy$outmostall #all outermost cH rows
yy$outmostfrequency #their frequency
#
###
#
data("wd3") #3D demo
df = wd3
yy = find_outmost_convexhull_points(df, kappas = 4)
yy$outmost #the rows of 4 outermost points
df[yy$outmost,] #the 4 outermost points
yy$outmostall #all outermost cH rows
yy$outmostfrequency #their frequency
```

---

find\_outmost\_partitioned\_convexhull\_points

*Function which finds the outermost convex hull points after making np samples and finding convex hull for each of them.*

---

**Description**

Function which finds the outermost convex hull points after making np samples and finding convex hull for each of them. To be used as initial solution in archetypal analysis

**Usage**

```
find_outmost_partitioned_convexhull_points(df, kappas, np = 10,
  nworkers = NULL)
```

**Arguments**

df	The data frame with dimensions n x d
kappas	The number of archetypes
np	The number of partitions that will be used (or the number of samples)
nworkers	The number of logical processors that will be used

**Value**

A list with members:

1. outmost, the first kappas most frequent outermost points as rows of data frame
2. outmostall, all the outermost points that have been found as rows of data frame
3. outmostfrequency, a matrix with frequency and cumulative frequency for outermost rows

**See Also**

[find\\_furthestsum\\_points](#), [find\\_outmost\\_projected\\_convexhull\\_points](#),  
[find\\_outmost\\_convexhull\\_points](#) & [find\\_outmost\\_points](#)

**Examples**

```
data("wd2") #2D demo
df = wd2
yy = find_outmost_partitioned_convexhull_points(df, kappas = 3, nworkers = 2)
yy$outmost #the rows of 3 outermost points
df[yy$outmost,] #the 3 outermost points
yy$outmostall #all outermost rows
yy$outmostfrequency #their frequency
```

---

find_outmost_points	<i>Function which finds the outermost points in order to be used as initial solution in archetypal analysis</i>
---------------------	---

---

**Description**

Function which finds the outermost points in order to be used as initial solution in archetypal analysis

**Usage**

```
find_outmost_points(df, kappas)
```

**Arguments**

df	The data frame with dimensions n x d
kappas	The number of archetypes

**Value**

A list with members:

1. outmost, the first kappas most frequent outermost points as rows of data frame
2. outmostall, all the outermost points that have been found as rows of data frame
3. outmostfrequency, a matrix with frequency and cumulative frequency for outermost rows



**Arguments**

df	The n x d data frame that will be used for Archetypal Analysis
kappas	The number of archetypes
npr	The dimension of the projected subspaces. It can be npr = 1 (then there are d such subspaces), or npr > 1 (then we have C(d,npr) different subspaces)
rseed	An integer to be used for the random seed if it will be necessary
doparallel	If it is set to TRUE, then parallel processing will be performed. That is absolutely required if n is very large and d>6.
nworkers	The number of logical processors that will be used for computing the projected convex hulls, which they are always C(d,npr).
uniquerows	If it is set to TRUE, then unique rows will be used for computing distance matrix and less resources will be needed.

**Details**

If npr = 1, then Convex Hull is identical with the range (min,max) for the relevant variable, otherwise the function uses the `chull` when npr = 2 and the `convhulln` for npr > 2. See [1] and [2] respectively for more details.

First all available projections are being considered and their Convex Hull are being computed. Then either the unique (if `uniquerows` = TRUE) or all (if `uniquerows` = FALSE) associated data rows form a matrix and finally by using `dist` we find the kappas most frequent outermost rows.

A special care is needed if the rows we have found are less than kappas. In that case, if a random sampling is necessary, the output `usedrandoms` informs us for the number of random rows and the `rseed` can be used for reproducibility.

**Value**

A list with members:

1. `outmost`, the first kappas most frequent outermost points as rows of data frame
2. `outmostall`, all the outermost points that have been found as rows of data frame
3. `outmostfrequency`, a matrix with frequency and cumulative frequency for outermost rows
4. `usedrandom`, an integer of randomly chosen rows, if it was necessary to complete the number of kappas rows
5. `chprojections`, all the Convex Hulls of the different C(d,npr) projections, i.e. the coordinate projection subspaces
6. `projected`, a data frame with rows the unique points that have been projected in order to create the relevant Convex Hulls of coordinate projection subspaces

**References**

- [1] Eddy, W. F. (1977). Algorithm 523: CONVEX, A new convex hull algorithm for planar sets. *ACM Transactions on Mathematical Software*, 3, 411-412. doi: 10.1145/355759.355768.
- [2] Barber, C.B., Dobkin, D.P., and Huhdanpraa, H.T., "The Quickhull algorithm for convex hulls" *ACM Trans. on Mathematical Software*, 22(4):469-483, Dec 1996, <http://www.qhull.org>

**See Also**

[find\\_furthestsum\\_points](#), [find\\_outmost\\_convexhull\\_points](#)  
[find\\_outmost\\_partitioned\\_convexhull\\_points](#) & [find\\_outmost\\_points](#)

**Examples**

```
#
data("wd2") #2D demo
df = wd2
yy = find_outmost_projected_convexhull_points(df, kappas = 3)
yy$outmost #the rows of 3 outmost projected convexhull points
yy$outmostall #all outmost found
yy$outmostfrequency #frequency table for all
yy$usedrandom #No random row was used
yy$chprojections #The Convex Hull of projection (one only here)
yy$projected #the 9 unique points that created the one only CH
df[yy$outmost,] #the 3 outmost projected convexhull points
#
###
#
data("wd3") #3D demo
df = wd3
yy = find_outmost_projected_convexhull_points(df, kappas = 4)
yy$outmost #the rows of 4 outmost projected convexhull points
yy$outmostall #all outmost found
yy$outmostfrequency #frequency table for all
yy$usedrandom #No random row was used
yy$chprojections #All the Convex Hulls of projections top coordinate planes
yy$projected #the 14 unique points that created all CHs
df[yy$outmost,] #the 4 outmost projected convexhull points
#
```

---

find\_pcha\_optimal\_parameters

*Finds the optimal updating parameters to be used for the PCHA algorithm*

---

**Description**

After creating a grid on the space of (mu\_up, mu\_down) it runs [archetypal](#) by using a given method & other running options passed by ellipsis (...) and finally finds those values which minimize the SSE at the end of testing\_iters iterations (default=10).

**Usage**

```
find_pcha_optimal_parameters(df, kappas, method = "projected_convexhull",
testing_iters = 10, nworkers = NULL, nprojected = 2, npartition = 10,
nfurthest = 100, sortrows = FALSE,
mup1 = 1.1, mup2 = 2.50, mdown1 = 0.1, mdown2 = 0.5, nmup = 10, nmdown = 10,
rseed = NULL, plot = FALSE, ...)
```

**Arguments**

df	The data frame with dimensions $n \times d$
kappas	The number of archetypes
method	The method that will be used for computing initial approximation: <ol style="list-style-type: none"> <li>1. projected_convexhull, see <a href="#">find_outmost_projected_convexhull_points</a></li> <li>2. convexhull, see <a href="#">find_outmost_convexhull_points</a></li> <li>3. partitioned_convexhull, see <a href="#">find_outmost_partitioned_convexhull_points</a></li> <li>4. furthestsum, see <a href="#">find_furthestsum_points</a></li> <li>5. outmost, see <a href="#">find_outmost_points</a></li> <li>6. random, a random set of kappas points will be used</li> </ol>
testing_iters	The maximum number of iterations to run for every pair ( $\mu_{up}$ , $\mu_{down}$ ) of parameters
nworkers	The number of logical processors that will be used for parallel computing (usually it is the double of available physical cores)
nprojected	The dimension of the projected subspace for <a href="#">find_outmost_projected_convexhull_points</a>
npartition	The number of partitions for <a href="#">find_outmost_partitioned_convexhull_points</a>
nfurthest	The number of times that <a href="#">FurthestSum</a> algorithm will be applied
sortrows	If it is TRUE, then rows will be sorted in <a href="#">find_furthestsum_points</a>
mup1	The minimum value of $\mu_{up}$ , default is 1.1
mup2	The maximum value of $\mu_{up}$ , default is 2.5
mdown1	The minimum value of $\mu_{down}$ , default is 0.1
mdown2	The maximum value of $\mu_{down}$ , default is 0.5
nmup	The number of points to be taken for $[\mu_{up1}, \mu_{up2}]$ , default is 10
nmdown	The number of points to be taken for $[\mu_{down1}, \mu_{down2}]$
rseed	The random seed that will be used for setting initial A matrix. Useful for reproducible results
plot	If it is TRUE, then a 3D plot for ( $\mu_{up}$ , $\mu_{down}$ , SSE) is created
...	Other arguments to be passed to function <a href="#">archetypal</a>

**Value**

A list with members:

1.  $\mu_{up\_opt}$ , the optimal found value for  $\mu_{Aup}$  and  $\mu_{Bup}$
2.  $\mu_{down\_opt}$ , the optimal found value for  $\mu_{Adown}$  and  $\mu_{Bdown}$
3.  $\min\_sse$ , the minimum SSE which corresponds to  $(\mu_{up\_opt}, \mu_{down\_opt})$
4.  $\text{seed\_used}$ , the used random seed, absolutely necessary for reproducing optimal results
5.  $\text{method\_used}$ , the method that was used for creating the initial solution
6.  $\text{sol\_initial}$ , the initial solution that was used for all grid computations
7.  $\text{testing\_iters}$ , the maximum number of iterations done by every grid computation



**See Also**[find\\_closer\\_points](#)**Examples**

```

{
  data("wd25")
  out = find_pcha_optimal_parameters(df = wd25, kappas = 5, rseed = 2020)
  # Time difference of 30.91101 secs
  # mu_up_opt mu_down_opt min_sse
  # 2.188889 0.100000 4.490980
  # Run now given the above optimal found parameters:
  aa = archetypal(df = wd25, kappas = 5,
                 initialrows = out$sol_initial, rseed = out$seed_used,
                 muAup = out$mu_up_opt, muAdown = out$mu_down_opt,
                 muBup = out$mu_up_opt, muBdown = out$mu_down_opt)
  aa[c("SSE", "varexpl", "iterations", "time" )]
  # $SSE
  # [1] 3.629542
  #
  # $varexpl
  # [1] 0.9998924
  #
  # $iterations
  # [1] 146
  #
  # $time
  # [1] 21.96
  # Compare it with a simple solution (time may vary)
  aa2 = archetypal(df = wd25, kappas = 5, rseed = 2020)
  aa2[c("SSE", "varexpl", "iterations", "time" )]
  # $SSE
  # [1] 3.629503
  #
  # $varexpl
  # [1] 0.9998924
  #
  # $iterations
  # [1] 164
  #
  # $time
  # [1] 23.55
  ## Of course the above was a "toy example", if your data has thousands or million rows,
  ## then the time reduction is much more conspicuous.
  # Close plot device:
  dev.off()
}

```

---

FurthestSum	<i>Application of FurthestSum algorithm in order to find an initial solution for Archetypal Analysis</i>
-------------	--

---

### Description

The FurthestSum algorithm as was written by Morup and Hansen in Matlab, see [1] and it is based on [2]. The algorithm has been converted in order to use commonly used data frames in R.

### Usage

```
FurthestSum(Y, kappas, irows, exclude = NULL)
```

### Arguments

Y	The data frame with dimensions $n \times d$
kappas	The number of archetypes
irows	The initially used rows of data frame for starting algorithm
exclude	The rows of data frame that we want to exclude from being checked

### Value

The vector of rows that constitute the initial FurthestSum solution

### References

- [1] Source: [https://www.mortenmorup.dk/MMhomepageUpdated\\_files/Page327.htm](https://www.mortenmorup.dk/MMhomepageUpdated_files/Page327.htm) , last accessed 2021-09-23
- [2] D.S. Hochbaum, D.B. Shmoys, A best possible heuristic for the k-center problem, Math. Oper. Res. 10(2) (1985) 180-184. <https://doi.org/10.1287/moor.10.2.180>

### See Also

[find\\_furthestsum\\_points](#)

### Examples

```
data("wd3") #3D demo
df = wd3
FurthestSum(df, kappas = 4, irows = sample(1:dim(df)[1],1))
```

---

gallupGPS6

*Gallup Global Preferences Study processed data set of six variables*

---

### Description

A 76132 x 6 data frame derived from Gallup Global Preferences Study, see [1] and [2] for details. It can be used as a big data set example.

### Usage

```
data("gallupGPS6")
```

### Format

A data frame with 76132 complete observations on the following 6 variables.

patience a numeric vector  
risktaking a numeric vector  
posrecip a numeric vector  
negrecip a numeric vector  
altruism a numeric vector  
trust a numeric vector

### Details

Data processing:

1. The non complete rows have been removed
2. The duplicated rows have also been removed

### Source

Individual data set was downloaded from <https://www.briq-institute.org/global-preferences/downloads>, last accessed 2019-12-17.

### References

[1] Falk, A., Becker, A., Dohmen, T., Enke, B., Huffman, D., & Sunde, U. (2018). Global evidence on economic preferences. *Quarterly Journal of Economics*, 133 (4), 1645-1692.

[2] Falk, A., Becker, A., Dohmen, T. J., Huffman, D., & Sunde, U. (2016). The preference survey module: A validated instrument for measuring risk, time, and social preferences. *IZA Discussion Paper No. 9674*.

**Examples**

```
data(gallupGPS6)
summary(gallupGPS6)
```

---

grouped\_resample      *Function for performing simple or Dirichlet resampling*

---

**Description**

The function may be used for standard bootstrapping or for subsampling, see [1]. This function allows samples to be drawn with or without replacement, by groups and with or without Dirichlet weights, see [2]. This provides a variety of options for researchers who wish to correct sample biases, estimate empirical confidence intervals, and/or subsample large data sets.

**Usage**

```
grouped_resample(in_data = NULL, grp_vector = NULL, grp_matrix = NULL,
                 replace = FALSE, option = "Simple", number_samples = 1,
                 nworkers = NULL, rseed = NULL)
```

**Arguments**

in_data	The initial data frame that must be re-sampled. It must contain: <ol style="list-style-type: none"> <li>1. an ID variable</li> <li>2. the variables of interest</li> <li>3. a grouping variable</li> </ol>
grp_vector	The grouping variable of the data frame, defined under the name 'group' for example
grp_matrix	A matrix that contains <ol style="list-style-type: none"> <li>1. the variable 'Group_ID' with entries all the available values of grouping variable</li> <li>2. the variable 'Resample_Size' with the sizes for each sample that will be created per grouping value</li> </ol>
replace	A logical input: TRUE/FALSE if replacement should be used or not, respectively
option	A character input with next possible values <ol style="list-style-type: none"> <li>1. "Simple", if we want to perform a simple re-sampling</li> <li>2. "Dirichlet", if we want to perform a Dirichlet weighted re-sampling</li> </ol>
number_samples	The number of samples to be created. If it is greater than one, then parallel processing is used.
nworkers	The number of logical processors that will be used for parallel computing (usually it is the double of available physical cores)
rseed	The random seed that will be used for sampling. Useful for reproducible results

**Value**

It returns a list of `number_samples` data frames with exactly the same variables as the initial one, except that group variable has now only the given value from input data frame.

**Author(s)**

David Midgley

**References**

- [1] D. N. Politis, J. P. Romano, M. Wolf, *Subsampling* (Springer-Verlag, New York, 1999).
- [2] Baath R (2018). *bayesboot: An Implementation of Rubin's (1981) Bayesian Bootstrap*. R package version 0.2.2, URL <https://CRAN.R-project.org/package=bayesboot>

**See Also**

[dirichlet\\_sample](#)

**Examples**

```
## Load absolute temperature data set:
data("AbsoluteTemperature")
df <- AbsoluteTemperature
## Find portions for climate zones
pcs <- table(df$z)/dim(df)[1]
## Choose the approximate size of the new sample and compute resample sizes
N <- round(sqrt(nrow(AbsoluteTemperature)))
resamplesizes=as.integer(round(N*pcs))
sum(resamplesizes)
## Create the grouping matrix
groupmat <- data.frame("Group_ID"=1:4,"Resample_Size"=resamplesizes)
groupmat
## Simple resampling:
resample_simple <- grouped_resample(in_data = df, grp_vector = "z",
                                   grp_matrix = groupmat, replace = FALSE, option = "Simple",
                                   number_samples = 1, nworkers = NULL, rseed = 20191220)
cat(dim(resample_simple[[1]]),"\n")
## Dirichlet resampling:
resample_dirichlet <- grouped_resample(in_data = df, grp_vector = "z",
                                       grp_matrix = groupmat, replace = FALSE, option = "Dirichlet",
                                       number_samples = 1, nworkers = NULL, rseed = 20191220)
cat(dim(resample_dirichlet[[1]]),"\n")
##
# ## Work in parallel and create many samples
# ## Choose a random seed
# nseed <- 20191119
# ## Simple
# reslist1 <- grouped_resample(in_data = df, grp_vector = "z", grp_matrix = groupmat,
#                               replace = FALSE, option = "Simple",
#                               number_samples = 10, nworkers = NULL,
#                               rseed = nseed)
```

```

# sapply(reslist1, dim)
# ## Dirichlet
# reslist2 <- grouped_resample(in_data = df, grp_vector = "z", grp_matrix = groupmat,
#                               replace = FALSE, option = "Dirichlet",
#                               number_samples = 10, nworkers = NULL,
#                               rseed = nseed)
# sapply(reslist2, dim)
# ## Check for same rows between 1st sample of 'Simple' and 1st sample of 'Dirichlet' ...
# mapply(function(x,y){sum(rownames(x)%in%rownames(y))},reslist1,reslist2)
#

```

---

kappa\_tools

---

*Compute kappa tools for data dimensionality analysis*


---

## Description

For a given data set and a given Archetypal Analysis (AA) solution, it finds a set of useful proxies for the dimensionality.

## Usage

```
kappa_tools(aa, df = NULL, numBins = 100, chvertices = NULL, verbose = FALSE, ...)
```

## Arguments

aa	An object of the class 'archetypal'
df	The data frame that was used for AA
numBins	The number of bins to be used for computing entropy
chvertices	The Convex Hull vertices, if they are given
verbose	Logical, set to TRUE if details must be printed
...	Other arguments, not used.

## Details

The ECDF for the Squared Errors (SE) is computed and then the relevant curve is classified as 'convex' or 'concave' and its UIK & inflection point is found. Then the number of used rows for creating archetypes is found. A procedure for creating BIC and adjusted BIC is used. Finally the percentage of used points that lie on the exact Convex Hull is given.

## Value

A list with next arguments:

ecdf	The ECDF of SE
Convexity	The convex or concave classification for ECDF curve
UIK	The UIK points of ECDF curve by using [1]

INFLECTION	The inflection points of ECDF curve by using [2]
NumberRowsUsed	The number of rows used for creating archetypes
RowsUsed	The exact rows used for creating archetypes
SSE	The Sum of SE
BIC	The computed BIC by using [3], [4]
adjBIC	The computed adjusted BIC by using [3], [4]
CXHE	The percentage of used points that lie on the exact Convex Hull

### Author(s)

Demetris T. Christopoulos, David F. Midgley (creator of BIC and adjBIC procedures)

### References

[1] Demetris T. Christopoulos, Introducing Unit Invariant Knee (UIK) As an Objective Choice for Elbow Point in Multivariate Data Analysis Techniques (March 1, 2016). Available at SSRN: <https://ssrn.com/abstract=3043076> or <http://dx.doi.org/10.2139/ssrn.3043076>

[2] Demetris T. Christopoulos, On the efficient identification of an inflection point, *International Journal of Mathematics and Scientific Computing*, (ISSN: 2231-5330), vol. 6(1), 2016.

[3] Felix Abramovich, Yoav Benjamini, David L. Donoho, Iain M. Johnstone. "Adapting to unknown sparsity by controlling the false discovery rate." *The Annals of Statistics*, 34(2) 584-653 April 2006. <https://doi.org/10.1214/009053606000000074>

[4] Murari, Andrea, Emmanuele Peluso, Francesco Cianfrani, Pasquale Gaudio, and Michele Lungaroni. 2019. "On the Use of Entropy to Improve Model Selection Criteria" *Entropy* 21, no. 4: 394. <https://doi.org/10.3390/e21040394>

### Examples

```
{
## Use the sample data "wd2"
data(wd2)
ch=convhulln(as.matrix(wd2), 'Fx')
chlist=as.list(ch)
chvertices = unique(do.call(c, chlist))
aa=archetypal(wd2, 3)
out=kappa_tools(aa , wd2, chvertices, verbose = T )
out
}
```

---

plot.archetypal      *Plot an object of the class archetypal.*

---

### Description

It makes a plot of the archetypes creating after using [archetypal](#)

### Usage

```
## S3 method for class 'archetypal'  
plot(x, ...)
```

### Arguments

x                    An object of the class archetypal  
...                   Other arguments (ignored)

### Details

If the data frame has column dimension at most 3, then a direct plot is available. Otherwise we use a "spike-spider" plot which is a combination of the common "spider" or "web" or "radar" plot with an additional "spike plot" that shows the frequency of each variable at the same line of the spider plot.

### Examples

```
{  
  ## Use the sample data "wd2"  
  data(wd2)  
  aa=archetypal(wd2, 3)  
  plot(aa)  
}
```

---

plot.kappa\_tools      *Plot an object of the class kappa\_tools*

---

### Description

It makes a plot of the results created after using [kappa\\_tools](#)

### Usage

```
## S3 method for class 'kappa_tools'  
plot(x, ...)
```



**Arguments**

x                    An object of the class kappa\_tools  
 ...                  Other arguments (ignored)

**Details**

A panel of 2 plots is being created, see [kappa\\_tools](#) for details.

**See Also**

[kappa\\_tools](#)

**Examples**

```
{
### Use the sample data "wd2"
data(wd2)
ch=convhulln(as.matrix(wd2),'Fx')
chlist=as.list(ch)
chvertices = unique(do.call(c,chlist))
aa=archetypal(wd2, 3)
out=kappa_tools(aa , wd2, chvertices, verbose = T )
plot(out)
}
```

---

plot.study\_AAconvergence

*Plot an object of the class study\_AAconvergence*

---

**Description**

It makes a plot of the results created after using [study\\_AAconvergence](#)

**Usage**

```
## S3 method for class 'study_AAconvergence'
plot(x, ...)
```

**Arguments**

x                    An object of the class study\_AAconvergence  
 ...                  Other arguments (ignored)

**Details**

A panel of 7 plots is being created, see [study\\_AAconvergence](#) for details.

**See Also**

[study\\_AAconvergence](#)

**Examples**

```
{
## Use the sample data "wd2"
data(wd2)
yy=study_AAconvergence(wd2, 3, plot = FALSE)
plot(yy)

}
```

---

plot\_archs

*A function for plotting archetypes*

---

**Description**

A data frame or matrix of archetypes can be plotted

**Usage**

```
plot_archs(archs, data = NULL, show_data = FALSE, ...)
```

**Arguments**

archs	The matrix or data frame of archetypes where each row represents an archetype
data	Optional argument, if used data frame is known
show_data	if it set to TRUE, then the used data frame will be plotted at the same plot
...	Other arguments (ignored)

**Details**

If the column dimension of the archetypes is less or equal to 3, then a normal plot is presented. Otherwise, a "spike-spider" plot is created, see [plot.archetypal](#) for details.

**See Also**

[plot.archetypal](#)

**Examples**

```
BY=matrix(c(5.430744, 2.043404, 3.128485, 3.146242, 2.710978, 4.781843), nrow = 3, byrow = TRUE)
plot_archs(BY)
```

---

print.archetypal	<i>Print an object of the class archetypal.</i>
------------------	---

---

### Description

It prints the output of [archetypal](#)

### Usage

```
## S3 method for class 'archetypal'  
print(x, ...)
```

### Arguments

x	An object of the class archetypal
...	Other arguments (ignored)

### Details

Since Archetypal Analysis (AA) is essentially one more matrix decomposition of the form  $Y \sim ABY$ , it is reasonable to print:

1. the  $kappas \times d$  matrix of archetypes found
2. the  $n \times kappas$  matrix A such that  $Y \sim ABY$  or Frobenius norm  $\|Y-ABY\|$  is minimum
3. the  $kappas \times n$  matrix B such that  $Y \sim ABY$  or Frobenius norm  $\|Y-ABY\|$  is minimum

### Examples

```
{  
## Use the sample data "wd2"  
data(wd2)  
aa=archetypal(wd2, 3)  
print(aa)  
}
```

---

study_AAconvergence	<i>Function which studies the convergence of Archetypal Analysis when using the PCHA algorithm</i>
---------------------	--

---

**Description**

First it finds an AA solution under given arguments while storing all iteration history (`save_history = TRUE`). Then it computes the LOWESS [1] of SSE and its relevant UIK point [2]. Study is performed for iterations after that point. The list of B-matrices and archetypes that were found are stored. The archetypes are being aligned, while the B-matrices are used for computing the used rows-weights, leading rows-weights and maybe percentage of used rows on Convex Hull. The Aitken SSE extrapolation plus the relevant error are computed. The order and rate of convergence are estimated. Finally a multi-plot panel is being created if asked.

**Usage**

```
study_AAconvergence(df, kappas, method = "projected_convexhull",
                    rseed = NULL, chvertices = NULL, plot = TRUE, ...)
```

**Arguments**

<code>df</code>	The data frame with dimensions $n \times d$
<code>kappas</code>	The number of archetypes
<code>method</code>	The method that will be used for computing initial approximation: <ol style="list-style-type: none"> <li>1. <code>projected_convexhull</code>, see <a href="#">find_outmost_projected_convexhull_points</a></li> <li>2. <code>convexhull</code>, see <a href="#">find_outmost_convexhull_points</a></li> <li>3. <code>partitioned_convexhull</code>, see <a href="#">find_outmost_partitioned_convexhull_points</a></li> <li>4. <code>furthestsum</code>, see <a href="#">find_furthestsum_points</a></li> <li>5. <code>outmost</code>, see <a href="#">find_outmost_points</a></li> <li>6. <code>random</code>, a random set of <code>kappas</code> points will be used</li> </ol>
<code>rseed</code>	The random seed that will be used for setting initial A matrix. Useful for reproducible results.
<code>chvertices</code>	The vector of rows which represents the vertices for Convex Hull (if available)
<code>plot</code>	If it is <code>TRUE</code> , then a panel of useful plots is created
<code>...</code>	Other arguments to be passed to function <a href="#">archetypal</a> , except <code>save_history</code> which must always be <code>TRUE</code>

**Details**

If we take natural logarithms at the next approximate equation

$$\epsilon_{n+1} = c\epsilon_n^p$$

for  $n = 1, 2, 3, \dots$ , then we'll find

$$\log(\epsilon_{n+1}) = \log(c) + p \log(\epsilon_n)$$

Thus a reasonable strategy for estimating order  $p$  and rate  $c$  is to perform a linear regression on above errors, after a selected iteration. That is the output of `order_estimation` and `rate_estimation`.

**Value**

A list with members:

1. SSE, a vector of all SSE from all AA iterations
2. SSE\_lowess, a vector of LOWESS values for SSE
3. UIK\_lowess, the UIK point [2] of SSE\_lowess
4. aitken, a data frame of Aitken [3] extrapolation and error for SSE after UIK\_lowess iteration
5. order\_estimation, the last term in estimating order of convergence, page 56 of [4], by using SSE after UIK\_lowess iteration
6. rate\_estimation, the last term in estimating rate of convergence, page 56 of [4], by using SSE after UIK\_lowess iteration
7. significance\_estimations, a data frame with standard errors and statistical significance for estimations
8. used\_on\_convexhull, the % of used rows which lie on Convex Hull (if given), as a sequence for iterations after UIK\_lowess one
9. aligned\_archetypes, the archetypes after UIK\_lowess iteration are being aligned by using [align\\_archetypes\\_from\\_list](#). The history of archetypes creation.
10. solution\_used, the AA output that has been used. Some times useful, especially for big data.

**References**

[1] Cleveland, W. S. (1979) Robust locally weighted regression and smoothing scatterplots. J. Amer. Statist. Assoc. 74, 829–836.

[2] Christopoulos, Demetris T., Introducing Unit Invariant Knee (UIK) As an Objective Choice for Elbow Point in Multivariate Data Analysis Techniques (March 1, 2016). Available at SSRN: <http://dx.doi.org/10.2139/ssrn.3043076>

[3] Aitken, A. "On Bernoulli's numerical solution of algebraic equations", Proceedings of the Royal Society of Edinburgh (1926) 46 pp. 289-305.

[4] Atkinson, K. E., An Introduction to Numerical Analysis, Wiley & Sons, 1989

**See Also**

[check\\_Bmatrix](#)

**Examples**

```
{
# Load data "wd2"
data(wd2)
ch = chull(wd2)
sa = study_AAconvergence(df = wd2, kappas = 3, rseed = 20191119,
                        verbose = FALSE, chvertices = ch)
names(sa)
```

```

# [1] "SSE"                "SSE_lowess"          "UIK_lowess"
# [4] "aitken"             "order_estimation"   "rate_estimation"
# [7] "significance_estimations" "used_on_convexhull" "aligned_archetypes"
# [10] "solution_used"
# sse=sa$SSE
# ssel=sa$SSE_lowess
sa$UIK_lowess
# [1] 36
# sa$aitken
sa$order_estimation
# [1] 1.007674
sa$rate_estimation
# [1] 0.8277613
sa$significance_estimations
#      estimation  std.error  t.value  p.value
# log(c) -0.1890305 0.014658947 -12.89523 5.189172e-12
# p      1.0076743 0.001616482 623.37475 3.951042e-50
# sa$used_on_convexhull
# sa$aligned_archetypes
data.frame(sa$solution_used[c("SSE", "varexpl", "iterations", "time")])
#      SSE  varexpl iterations time
# 1 1.717538 0.9993186        62 8.39
# Close plot device:
dev.off()

}

```

---

summary.archetypal      *Summary for an object of the class archetypal.*

---

## Description

It gives a summary for the output of [archetypal](#)

## Usage

```
## S3 method for class 'archetypal'
summary(object, ...)
```

## Arguments

object	An object of the class archetypal
...	Other arguments (ignored)

## Details

Next info is given:

1. the number of observations or the row number of the data frame

2. the dimension of the data variables
3. the number of archetypes that was used
4. the computed archetypes
5. a vector of run details: SSE, VarianceExplained, Convergence, Iterations, EllapsedTime
6. the calling command

### Examples

```
{
## Use the sample data "wd2"
data(wd2)
aa=archetypal(wd2, 3)
summary(aa)
}
```

---

wd2

*2D data set for demonstration purposes*

---

### Description

A data frame of 100 2D points

### Usage

```
data("wd2")
```

### Format

matrix 100 x 2

### Examples

```
# Creation of data set "wd2" from 3 corner-points:
p1 = c(1,2);p2 = c(3,5);p3 = c(7,3)
dp = rbind(p1,p2,p3);dp
set.seed(9102)
pts = t(sapply(1:100, function(i,dp){
  cc = runif(3)
  cc = cc/sum(cc)
  colSums(dp*cc)
},dp))
df = data.frame(pts)
colnames(df) = c("x","y")
head(df)
# Check all equal:
data(wd2)
all.equal(wd2,df)
# [1] TRUE
```

wd25

*2D data set created by 5 points for demonstration purposes***Description**

A data frame of 600 2D points

**Usage**

```
data("wd25")
```

**Format**

matrix 600 x 2

**Examples**

```
# Creation of data set "wd25" from 5 corner points:
set.seed(20191119)
p1 = c(3,2);p2 = c(4,6);p3 = c(7,8)
p4 = c(9,4);p5 = c(6,1)
dp = rbind(p1,p2,p3,p4,p5)
colnames(dp) = c('x','y')
pts=lapply(1:150, function(i,dp){
  c0 = runif(dim(dp)[1]);c0 = c0/sum(c0);pt0 = colSums(dp*c0)
  c1 = runif(3);c1 = c1/sum(c1);pt1 = colSums(dp[1:3,]*c1)
  c2 = runif(3);c2 = c2/sum(c2);pt2 = colSums(dp[c(4,5,1),]*c2)
  c3 = runif(3);c3 = c3/sum(c3);pt3 = colSums(dp[2:4,]*c3)
  rbind(pt0,pt1,pt2,pt3)
},dp)
df = do.call(rbind,pts)
rownames(df) = 1:dim(df)[1]
head(df)
# Check all equal
data("wd25")
all.equal(df,wd25)
# [1] TRUE
```

wd3

*3D data set for demonstration purposes***Description**

A data frame of 100 3D points

**Usage**

```
data("wd3")
```



**Format**

matrix 100 x 3

**Examples**

```
# Creation of data set "wd3" from 4 corner points:
p1 = c(3,0,0);p2 = c(0,5,0)
p3 = c(3,5,7);p4 = c(0,0,0)
# The data frame of generators
dp = data.frame(rbind(p1,p2,p3,p4))
colnames(dp) = c("x","y","z")
dp = dp[chull(dp),]
set.seed(9102)
df = data.frame(t(sapply(1:100, function(i,dp){
  cc = runif(4)
  cc = cc/sum(cc)
  colSums(dp*cc)
},dp)))
colnames(df) = c("x","y","z")
head(df)
# Check all.equal to "wd3"
data(wd3)
all.equal(df,wd3)
# [1] TRUE
```

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