

# Package ‘MixGHD’

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**Type** Package

**Title** Model Based Clustering, Classification and Discriminant Analysis  
Using the Mixture of Generalized Hyperbolic Distributions

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**Description** Carries out model-based clustering, classification and discriminant analysis using five different models. The models are all based on the generalized hyperbolic distribution. The first model 'MGHD' (Browne and McNicholas (2015) <doi:10.1002/cjs.11246>) is the classical mixture of generalized hyperbolic distributions. The 'MGHFA' (Tortora et al. (2016) <doi:10.1007/s11634-015-0204-z>) is the mixture of generalized hyperbolic factor analyzers for high dimensional data sets. The 'MS-GHD' is the mixture of multiple scaled generalized hyperbolic distributions, the 'cMS-GHD' is a 'MSGHD' with convex contour plots and the 'MCGHD', mixture of coalesced generalized hyperbolic distributions is a new more flexible model (Tortora et al. (2019)<doi:10.1007/s00357-019-09319-3>. The paper related to the software can be found at <doi:10.18637/jss.v098.i03>.

**Imports** Bessel,stats, mvtnorm, ghyp, numDeriv, mixture, e1071,cluster, methods

**Depends** MASS, R (>= 3.1.3)

**NeedsCompilation** yes

**License** GPL (>= 2)

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**SystemRequirements** GNU make

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ARI *Adjusted Rand Index.*

---

**Description**

Compares two classifications using the adjusted Rand index (ARI).

**Usage**

ARI(x=NULL, y=NULL)

**Arguments**

x                    A n dimensional vector of class labels.  
y                    A n dimensional vector of class labels. .

**Details**

The ARI has expected value 0 in case of random partition, it is equal to one in case of perfect agreement..

**Value**

The adjusted Rand index value

**Author(s)**

Cristina Tortora Maintainer: Cristina Tortora <crisrina.tortora@sjsu.edu>

**References**

L. Hubert and P. Arabie (1985) Comparing Partitions, *Journal of the Classification* 2:193-218.

**Examples**

```
##loading banknote data
data(banknote)

##model estimation
res=MGHD(data=banknote[,2:7], G=2 )

#result
ARI(res@map, banknote[,1])
```

---

banknote	<i>Swiss Banknote data</i>
----------	----------------------------

---

**Description**

The data set contain 6 measures of 100 genuine and 100 counterfeit Swiss franc banknotes.

**Usage**

```
data(banknote)
```

**Format**

A data frame with the following variables:

**Status** the status of the banknote: genuine or counterfeit

**Length** Length of bill (mm)

**Left** Width of left edge (mm)

**Right** Width of right edge (mm)

**Bottom** Bottom margin width (mm)

**Top** Top margin width (mm)

**Diagonal** Length of diagonal (mm)

**References**

Flury, B. and Riedwyl, H. (1988). *Multivariate Statistics: A practical approach*. London: Chapman & Hall, Tables 1.1 and 1.2, pp. 5-8

---

bankruptcy	<i>Bankruptcy data</i>
------------	------------------------

---

**Description**

The data set contain the ratio of retained earnings (RE) to total assets, and the ratio of earnings before interests and taxes (EBIT) to total assets of 66 American firms recorded in the form of ratios. Half of the selected firms had filed for bankruptcy.

**Usage**

```
data(bankruptcy)
```

**Format**

A data frame with the following variables:

**Y** the status of the firm: 0 bankruptcy or 1 financially sound.

**RE** ratio

**EBIT** ratio

**References**

Altman E.I. (1968). Financial ratios, discriminant analysis and the prediction of corporate bankruptcy. *J Finance* **23**(4): 589-609

---

cMSGHD	<i>Convex mixture of multiple scaled generalized hyperbolic distributions (cMSGHD).</i>
--------	---

---

**Description**

Carries out model-based clustering using the convex mixture of multiple scaled generalized hyperbolic distributions. The cMSGHD only allows convex level sets.

**Usage**

```
cMSGHD(data=NULL, gpar0=NULL, G=2, max.iter=100, label=NULL, eps=1e-2,  
method="km", scale=TRUE, nr=10, modelSel="AIC")
```

**Arguments**

data	A n x p matrix or data frame such that rows correspond to observations and columns correspond to variables.
gpar0	(optional) A list containing the initial parameters of the mixture model. See the 'Details' section.
G	The range of values for the number of clusters.
max.iter	(optional) A numerical parameter giving the maximum number of iterations each EM algorithm is allowed to use.
label	( optional) A n dimensional vector, if label[i]=k then observation i belongs to group k, if NULL then the data has no known groups.
eps	(optional) A number specifying the epsilon value for the convergence criteria used in the EM algorithms. For each algorithm, the criterion is based on the difference between the log-likelihood at an iteration and an asymptotic estimate of the log-likelihood at that iteration. This asymptotic estimate is based on the Aitken acceleration.
method	( optional) A string indicating the initialization criterion, if not specified kmeans clustering is used. Alternative methods are: hierarchical "hierarchical", random "random", kmedoids "kmedoids", and model based "modelBased"
scale	( optional) A logical value indicating whether or not the data should be scaled, true by default.
nr	( optional) A number indicating the number of starting value when random is used, 10 by default.
modelSel	( optional) A string indicating the model selection criterion, if not specified AIC is used. Alternative methods are: BIC,ICL, and AIC3

**Details**

The arguments gpar0, if specified, is a list structure containing at least one p dimensional vector mu, alpha and phi, a pxp matrix gamma, and a px2 matrix cpl containing the vector omega and the vector lambda.

**Value**

A S4 object of class [MixGHD](#) with slots:

index	Value of the index used for model selection (AIC or ICL or BIC or AIC3) for each G,the index used is specified by the user, if not specified AIC is used.
BIC	Bayesian information criterion.
ICL	Integrated completed likelihood.
AIC	Akaike information criterion.
AIC3	Akaike information criterion 3.
gpar	A list of the model parameters
loglik	The log-likelihood values.
map	A vector of integers indicating the maximum a posteriori classifications for the best model.
z	A matrix giving the raw values upon which map is based.

**Author(s)**

Cristina Tortora, Aisha ElSherbiny, Ryan P. Browne, Brian C. Franczak, and Paul D. McNicholas.  
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**References**

C. Tortora, B.C. Franczak, R.P. Browne, and P.D. McNicholas (2019). A Mixture of Coalesced Generalized Hyperbolic Distributions. *Journal of Classification* 36(1) 26-57.  
 C. Tortora, R. P. Browne, A. ElSherbiny, B. C. Franczak, and P. D. McNicholas (2021). Model-Based Clustering, Classification, and Discriminant Analysis using the Generalized Hyperbolic Distribution: MixGHD R package, *Journal of Statistical Software* 98(3) 1–24, <doi:10.18637/jss.v098.i03>.

**See Also**

[MGHD MSGHD](#)

**Examples**

```
##Generate random data
set.seed(3)

mu1 <- mu2 <- c(0,0)
Sigma1 <- matrix(c(1,0.85,0.85,1),2,2)
Sigma2 <- matrix(c(1,-0.85,-0.85,1),2,2)

X1 <- mvrnorm(n=150,mu=mu1,Sigma=Sigma1)
X2 <- mvrnorm(n=150,mu=mu2,Sigma=Sigma2)

X <- rbind(X1,X2)

##model estimation
em=cMSGHD(X,G=2,max.iter=30,method="random",nr=2)

#result
plot(em)
```

---

 coef

*Coefficients for objects of class MixGHD*

---

**Description**

Coefficients of the estimated model.

**Usage**

```
## S4 method for signature 'MixGHD'  
coef(object)
```

**Arguments**

object            An S4 object of class [MixGHD](#).

**Value**

The coefficients of the estimated model

**Author(s)**

Cristina Tortora Maintainer: Cristina Tortora <crisrina.tortora@sjtu.edu>

**Examples**

```
##loading bankruptcy data  
data(bankruptcy)  
  
##model estimation  
res=MCGHD(data=bankruptcy[,2:3],G=2,method="kmedoids",max.iter=30)  
#rcoefficients of the model  
coef(res)
```

---

contourpl

*Contour plot*

---

**Description**

Contour plot for a given set of parameters.

**Usage**

```
contourpl(input)
```

**Arguments**

input            An S4 object of class [MixGHD](#).

**Value**

The contour plot

**Author(s)**

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**Examples**

```
##loading bankruptcy data
data(bankruptcy)

##model estimation
res=MCGHD(data=bankruptcy[,2:3],G=2,method="kmedoids",max.iter=30)
#result
contourpl(res)
```

---

DA	<i>Discriminant analysis using the mixture of generalized hyperbolic distributions.</i>
----	---

---

**Description**

Carries out model-based discriminant analysis using 5 different models: the mixture of multiple scaled generalized hyperbolic distributions (MGHD), the mixture of generalized hyperbolic factor analyzers (MGHFA), the mixture of multiple scaled generalized hyperbolic distributions (MSGHD), the mixture of convex multiple scaled generalized hyperbolic distributions (cMSGHD) and the mixture of coalesed generalized hyperbolic distributions (MCGHD).

**Usage**

```
DA(train,trainL,test,testL,method="MGHD",starting="km",max.iter=100,
eps=1e-2,q=2,scale=TRUE)
```

**Arguments**

train	A n1 x p matrix or data frame such that rows correspond to observations and columns correspond to variables of the training data set.
trainL	A n1 dimensional vector of membership for the units of the training set. If trainL[i]=k then observation belongs to group k.
test	A n2 x p matrix or data frame such that rows correspond to observations and columns correspond to variables of the test data set.
testL	A n2 dimensional vector of membership for the units of the test set. If testL[i]=k then observation belongs to group k.
method	( optional) A string indicating the method to be used form discriminant analysis , if not specified MGHD is used. Alternative methods are: MGHFA, MSGHD, cMSGHD, MCGHD.
starting	( optional) A string indicating the initialization criterion, if not specified kmeans clustering is used. Alternative methods are: hierarchical "hierarchical",random "random", kmedoids "kmedoids", and model based "modelBased"

max.iter	(optional) A numerical parameter giving the maximum number of iterations each EM algorithm is allowed to use.
eps	(optional) A number specifying the epsilon value for the convergence criteria used in the EM algorithms. For each algorithm, the criterion is based on the difference between the log-likelihood at an iteration and an asymptotic estimate of the log-likelihood at that iteration. This asymptotic estimate is based on the Aitken acceleration.
q	(optional) used only if MGHFA method is selected. A numerical parameter giving the number of factors.
scale	( optional) A logical value indicating whether or not the data should be scaled, true by default.

### Value

A list with components

model	An S4 object of class <code>MixGHD</code> with the model parameters.
testMembership	A vector of integers indicating the membership of the units in the test set
ARITest	A value indicating the adjusted rand index for the test set.
ARITrain	A value indicating the adjusted rand index for the train set.

### Author(s)

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### References

- R.P. Browne, and P.D. McNicholas (2015). A Mixture of Generalized Hyperbolic Distributions. *Canadian Journal of Statistics*, 43.2 176-198.
- C. Tortora, B.C. Franczak, R.P. Browne, and P.D. McNicholas (2019). A Mixture of Coalesced Generalized Hyperbolic Distributions. *Journal of Classification* (to appear).
- C.Tortora, P.D. McNicholas, and R.P. Browne (2016). Mixtures of Generalized Hyperbolic Factor Analyzers. *Advanced in data analysis and classification* 10(4) p.423-440.

### See Also

"`MixGHD`" `MGHD` `MGHFA` `MSGHD` `MSGHD` `MCGHD` `ARI` `MixGHD-class` `MixGHD`

### Examples

```
##loading banknote data
data(banknote)
banknote[,1]=as.numeric(factor(banknote[,1]))

##divide the data in training set and test set
train=banknote[c(1:74,126:200),]
test=banknote[75:125,]
```

```
##model estimation
model=DA(train[,2:7],train[,1],test[,2:7],test[,1],method="MGHD",max.iter=20)

#result
model$ARItest
```

---

dCGHD

*Density of a coalesced generalized hyperbolic distribution (MSGHD).*


---

### Description

Compute the density of a p dimensional coalesced generalized hyperbolic distribution.

### Usage

```
dCGHD(data,p,mu=rep(0,p),alpha=rep(0,p),sigma=diag(p),lambda=1,omega=1,
omegav=rep(1,p),lambdav=rep(1,p),wg=0.5,gam=NULL,phi=NULL)
```

### Arguments

data	n x p data set
p	number of variables.
mu	(optional) the p dimensional mean
alpha	(optional) the p dimensional skewness parameter alpha
sigma	(optional) the p x p dimensional scale matrix
lambda	(optional) the 1 dimensional index parameter lambda
omega	(optional) the 1 dimensional concentration parameter omega
omegav	(optional) the p dimensional concentration parameter omega
lambdav	(optional) the p dimensional index parameter lambda
wg	(optional) weight
gam	(optional) the p x p gamma matrix
phi	(optional) the p dimensional vector phi

### Details

The default values are: 0 for the mean and the skewness parameter alpha,  $\text{diag}(p)$  for sigma, 1 for omega, and 0.5 for lambda.

### Value

A n dimensional vector with the density from a coalesced generalized hyperbolic distribution

**Author(s)**

Cristina Tortora, Aisha ElSherbiny, Ryan P. Browne, Brian C. Franczak, and Paul D. McNicholas.  
 Maintainer: Cristina Tortora <cristina.tortora@sjsu.edu>

**References**

C. Tortora, B.C. Franczak, R.P. Browne, and P.D. McNicholas (2019). A Mixture of Coalesced Generalized Hyperbolic Distributions. *Journal of Classification* (to appear).

**Examples**

```
x = seq(-3,3,length.out=30)
y = seq(-3,3,length.out=30)
xyS1 = matrix(0,nrow=length(x),ncol=length(y))
for(i in 1:length(x)){
  for(j in 1:length(y)){
    xy <- matrix(cbind(x[i],y[j]),1,2)
    xyS1[i,j] = dCGHD(xy,2)
  }
}
contour(x=x,y=y,z=xyS1, levels=c(.005,.01,.025,.05,.1,.25), main="CGHD",ylim=c(-3,3), xlim=c(-3,3))
```

---

dGHD

*Density of a generalized hyperbolic distribution (GHD).*


---

**Description**

Compute the density of a p dimensional generalized hyperbolic distribution.

**Usage**

```
dGHD(data,p, mu=rep(0,p),alpha=rep(0,p),sigma=diag(p),omega=1,lambda=0.5, log=FALSE)
```

**Arguments**

data	n x p data set
p	number of variables.
mu	(optional) the p dimensional mean
alpha	(optional) the p dimensional skewness parameter alpha
sigma	(optional) the p x p dimensional scale matrix
omega	(optional) the unidimensional concentration parameter omega
lambda	(optional) the unidimensional index parameter lambda
log	(optional) if TRUE returns the log of the density

**Details**

The default values are: 0 for the mean and the skewness parameter  $\alpha$ ,  $\text{diag}(p)$  for  $\sigma$ , 1 for  $\omega$ , and 0.5 for  $\lambda$ .

**Value**

A  $n$  dimensional vector with the density from a generalized hyperbolic distribution

**Author(s)**

Cristina Tortora, Aisha ElSherbiny, Ryan P. Browne, Brian C. Franczak, and Paul D. McNicholas.  
Maintainer: Cristina Tortora <crisrina.tortora@sjsu.edu>

**References**

R.P. Browne, and P.D. McNicholas (2015). A Mixture of Generalized Hyperbolic Distributions. *Canadian Journal of Statistics*, 43.2 176-198

**Examples**

```
x = seq(-3,3,length.out=50)
y = seq(-3,3,length.out=50)
xyS1 = matrix(0,nrow=length(x),ncol=length(y))
for(i in 1:length(x)){
  for(j in 1:length(y)){
    xy <- matrix(cbind(x[i],y[j]),1,2)
    xyS1[i,j] = dGHD(xy,2)
  }
}
contour(x=x,y=y,z=xyS1, levels=c(.005, .01, .025, .05, .1, .25), main="MGHD",ylim=c(-3,3), xlim=c(-3,3))
```

---

dMSGHD

*Density of a multiple-scaled generalized hyperbolic distribution (MSGHD).*

---

**Description**

Compute the density of a  $p$  dimensional multiple-scaled generalized hyperbolic distribution.

**Usage**

```
dMSGHD(data,p,mu=rep(0,p),alpha=rep(0,p),sigma=diag(p),omegav=rep(1,p),
        lambdav=rep(0.5,p),gam=NULL,phi=NULL,log=FALSE)
```

**Arguments**

data	n x p data set
p	number of variables.
mu	(optional) the p dimensional mean
alpha	(optional) the p dimensional skewness parameter alpha
sigma	(optional) the p x p dimensional scale matrix
omegav	(optional) the p dimensional concentration parameter omega
lambdav	(optional) the p dimensional index parameter lambda
gam	(optional) the p x p gamma matrix
phi	(optional) the p dimensional vector phi
log	(optional) if TRUE returns the log of the density

**Details**

The default values are: 0 for the mean and the skewness parameter alpha,  $\text{diag}(p)$  for sigma, 1 for omega, and 0.5 for lambda.

**Value**

A n dimensional vector with the density from a multiple-scaled generalized hyperbolic distribution

**Author(s)**

Cristina Tortora, Aisha ElSherbiny, Ryan P. Browne, Brian C. Franczak, and Paul D. McNicholas.  
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**References**

C. Tortora, B.C. Franczak, R.P. Browne, and P.D. McNicholas (2019). A Mixture of Coalesced Generalized Hyperbolic Distributions. *Journal of Classification* (to appear).

**Examples**

```
x = seq(-3,3,length.out=50)
y = seq(-3,3,length.out=50)
xyS1 = matrix(0,nrow=length(x),ncol=length(y))
for(i in 1:length(x)){
  for(j in 1:length(y)){
    xy <- matrix(cbind(x[i],y[j]),1,2)
```

```

        xyS1[i, j] = dMSGHD(xy, 2)
    }
}
contour(x=x, y=y, z=xyS1, levels=seq(.005, .25, by=.005), main="MSGHD")

```

---

MCGHD

*Mixture of coalesced generalized hyperbolic distributions (MCGHD).*


---

### Description

Carries out model-based clustering using the mixture of coalesced generalized hyperbolic distributions.

### Usage

```

MCGHD(data=NULL, gpar0=NULL, G=2, max.iter=100, eps=1e-2, label=NULL,
method="km", scale=TRUE, nr=10, modelSel="AIC")

```

### Arguments

data	A n x p matrix or data frame such that rows correspond to observations and columns correspond to variables.
gpar0	(optional) A list containing the initial parameters of the mixture model. See the 'Details' section.
G	The range of values for the number of clusters.
max.iter	(optional) A numerical parameter giving the maximum number of iterations each EM algorithm is allowed to use.
eps	(optional) A number specifying the epsilon value for the convergence criteria used in the EM algorithms. For each algorithm, the criterion is based on the difference between the log-likelihood at an iteration and an asymptotic estimate of the log-likelihood at that iteration. This asymptotic estimate is based on the Aitken acceleration.
label	( optional) A n dimensional vector, if label[i]=k then observation i belongs to group k, If label[i]=0 then observation i has no known group, if NULL then the data has no known groups.
method	( optional) A string indicating the initialization criterion, if not specified kmeans clustering is used. Alternative methods are: hierarchical "hierarchical", random "random", and model based "modelBased"
scale	( optional) A logical value indicating whether or not the data should be scaled, true by default.
nr	( optional) A number indicating the number of starting value when random is used, 10 by default.
modelSel	( optional) A string indicating the model selection criterion, if not specified AIC is used. Alternative methods are: BIC, ICL, and AIC3

**Details**

The arguments `gpar0`, if specified, has to be a list structure containing as much element as the number of components `G`. Each element must include the following parameters: one `p` dimensional vector `mu`, `alpha` and `phi`, a `pxp` matrix `gamma`, a `px2` vector `cpl` containing the vectors `omega` and `lambda`, and a 2-dimensional vector containing the `omega0` and `lambda0`.

**Value**

A S4 object of class `MixGHD` with slots:

<code>index</code>	Value of the index used for model selection (AIC or ICL or BIC or AIC3) for each <code>G</code> , the index used is specified by the user, if not specified AIC is used.
<code>BIC</code>	Bayesian information criterion.
<code>ICL</code>	Integrated completed likelihood..
<code>AIC</code>	Akaike information criterion.
<code>AIC3</code>	Akaike information criterion 3.
<code>gpar</code>	A list of the model parameters in the rotated space.
<code>loglik</code>	The log-likelihood values.
<code>map</code>	A vector of integers indicating the maximum a posteriori classifications for the best model.
<code>par</code>	A list of the model parameters.
<code>z</code>	A matrix giving the raw values upon which <code>map</code> is based.

**Author(s)**

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**References**

C. Tortora, B.C. Franczak, R.P. Browne, and P.D. McNicholas (2019). A Mixture of Coalesced Generalized Hyperbolic Distributions. *Journal of Classification* 36(1) 26-57. \ C. Tortora, R. P. Browne, A. ElSherbiny, B. C. Franczak, and P. D. McNicholas (2021). Model-Based Clustering, Classification, and Discriminant Analysis using the Generalized Hyperbolic Distribution: MixGHD R package, *Journal of Statistical Software* 98(3) 1–24, <doi:10.18637/jss.v098.i03>.

**See Also**

[MGHD](#), [MSGHD](#)

**Examples**

```
##loading banknote data
data(banknote)

##model estimation
model=MCGHD(banknote[,2:7],G=2,max.iter=20)
```

```
#result
#summary(model)
#plot(model)
table(banknote[,1],model@map)
```

---

MGHD

*Mixture of generalized hyperbolic distributions (MGHD).*


---

### Description

Carries out model-based clustering and classification using the mixture of generalized hyperbolic distributions.

### Usage

```
MGHD(data=NULL, gpar0=NULL, G=2, max.iter=100, label=NULL, eps=1e-2,
method="kmeans", scale=TRUE, nr=10, modelSel="AIC")
```

### Arguments

data	A n x p matrix or data frame such that rows correspond to observations and columns correspond to variables.
gpar0	(optional) A list containing the initial parameters of the mixture model. See the 'Details' section.
G	The range of values for the number of clusters.
max.iter	(optional) A numerical parameter giving the maximum number of iterations each EM algorithm is allowed to use.
label	( optional) A n dimensional vector, if label[i]=k then observation i belongs to group k, If label[i]=0 then observation i has no known group, if NULL then the data has no known groups.
eps	(optional) A number specifying the epsilon value for the convergence criteria used in the EM algorithms. For each algorithm, the criterion is based on the difference between the log-likelihood at an iteration and an asymptotic estimate of the log-likelihood at that iteration. This asymptotic estimate is based on the Aitken acceleration.
method	( optional) A string indicating the initialization criterion, if not specified kmeans clustering is used. Alternative methods are: hierarchical "hierarchical", random "random", and model based "modelBased" clustering
scale	( optional) A logical value indicating whether or not the data should be scaled, true by default.
nr	( optional) A number indicating the number of starting value when random is used, 10 by default.
modelSel	( optional) A string indicating the model selection criterion, if not specified AIC is used. Alternative methods are: BIC, ICL, and AIC3

**Details**

The arguments `gpar0`, if specified, is a list structure containing at least one  $p$  dimensional vector `mu`, and `alpha`, a  $p \times p$  matrix `sigma`, and a 2 dimensional vector containing `omega` and `lambda`.

**Value**

A S4 object of class `MixGHD` with slots:

<code>index</code>	Value of the index used for model selection (AIC or ICL or BIC or AIC3) for each G, the index used is specified by the user, if not specified AIC is used.
<code>BIC</code>	Bayesian information criterion.
<code>ICL</code>	Integrated completed likelihood..
<code>AIC</code>	Akaike information criterion.
<code>AIC3</code>	Akaike information criterion 3.
<code>gpar</code>	A list of the model parameters.
<code>loglik</code>	The log-likelihood values.
<code>map</code>	A vector of integers indicating the maximum a posteriori classifications for the best model.
<code>z</code>	A matrix giving the raw values upon which <code>map</code> is based.

**Author(s)**

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Maintainer: Cristina Tortora <cristina.tortora@sjsu.edu>

**References**

R.P. Browne, and P.D. McNicholas (2015). A Mixture of Generalized Hyperbolic Distributions. *Canadian Journal of Statistics*, 43.2 176-198\ C. Tortora, R. P. Browne, A. ElSherbiny, B. C. Franczak, and P. D. McNicholas (2021). Model-Based Clustering, Classification, and Discriminant Analysis using the Generalized Hyperbolic Distribution: MixGHD R package, *Journal of Statistical Software* 98(3) 1–24, <doi:10.18637/jss.v098.i03>.

**Examples**

```
##loading crabs data
data(crabs)

##model estimation
model=MGHD(data=crabs[,4:8], G=2 )

#result
plot(model)
table(model@map, crabs[,2])

## Classification
##loading bankruptcy data
data(bankruptcy)
```

```

#70% belong to the training set
label=bankruptcy[,1]
#for a Classification porpuse the label cannot be 0
label[1:33]=2
a=round(runif(20)*65+1)
label[a]=0

##model estimation
model=MGHD(data=bankruptcy[,2:3], G=2, label=label )

#result
table(model@map,bankruptcy[,1])
plot(model)

```

---

MGHFA

*Mixture of generalized hyperbolic factor analyzers (MGHFA).*


---

### Description

Carries out model-based clustering and classification using the mixture of generalized hyperbolic factor analyzers.

### Usage

```

MGHFA(data=NULL, gpar0=NULL, G=2, max.iter=100,
label =NULL ,q=2,eps=1e-2 , method="kmeans", scale=TRUE ,nr=10)

```

### Arguments

data	A matrix or data frame such that rows correspond to observations and columns correspond to variables.
gpar0	(optional) A list containing the initial parameters of the mixture model. See the 'Details' section.
G	The range of values for the number of clusters.
max.iter	(optional) A numerical parameter giving the maximum number of iterations each EM algorithm is allowed to use.
label	( optional) A n dimensional vector, if label[i]=k then observation i belongs to group k, If label[i]=0 then observation i has no known group, if NULL then the data has no known groups.
q	The range of values for the number of factors.
eps	(optional) A number specifying the epsilon value for the convergence criteria used in the EM algorithms. For each algorithm, the criterion is based on the difference between the log-likelihood at an iteration and an asymptotic estimate of the log-likelihood at that iteration. This asymptotic estimate is based on the Aitken acceleration.

method	( optional) A string indicating the initialization criterion, if not specified kmeans clustering is used. Alternative methods are: hierarchical "hierarchical" and model based "modelBased" clustering
scale	( optional) A logical value indicating whether or not the data should be scaled, true by default.
nr	( optional) A number indicating the number of starting value when random is used, 10 by default.

### Details

The arguments `gpar0`, if specified, is a list structure containing at least one  $p$  dimensional vector `mu`, `alpha` and `phi`, a  $p \times p$  matrix `gamma`, a 2 dimensional vector `cpl` containing `omega` and `lambda`.

### Value

A S4 object of class `MixGHD` with slots:

Index	Bayesian information criterion value for each combination of $G$ and $q$ .
BIC	Bayesian information criterion.
<code>gpar</code>	A list of the model parameters.
<code>loglik</code>	The log-likelihood values.
<code>map</code>	A vector of integers indicating the maximum a posteriori classifications for the best model.
<code>z</code>	A matrix giving the raw values upon which <code>map</code> is based.

### Author(s)

Cristina Tortora, Aisha ElSherbiny, Ryan P. Browne, Brian C. Franczak, and Paul D. McNicholas.  
Maintainer: Cristina Tortora <crisrina.tortora@sjsu.edu>

### References

C.Tortora, P.D. McNicholas, and R.P. Browne (2016). Mixtures of Generalized Hyperbolic Factor Analyzers. *Advanced in data analysis and classification* 10(4) p.423-440. C. Tortora, R. P. Browne, A. ElSherbiny, B. C. Franczak, and P. D. McNicholas (2021). Model-Based Clustering, Classification, and Discriminant Analysis using the Generalized Hyperbolic Distribution: MixGHD R package, *Journal of Statistical Software* 98(3) 1–24, <doi:10.18637/jss.v098.i03>.

### Examples

```
## Classification
#70% belong to the training set
data(sonar)
label=sonar[,61]
set.seed(4)
a=round(runif(62)*207+1)
label[a]=0
```

```
##model estimation
model=MGHFA(data=sonar[,1:60], G=2, max.iter=25 ,q=2,label=label)

#result
table(model@map,sonar[,61])
summary(model)
```

---

 MixGHD class

 Class "MixGHD"
 

---

### Description

This class pertains to results of the application of function [MGHD](#), [MSGHD](#), [cMSGHD](#), [MCGHD](#), and [MGHFA](#).

### Objects from the Class

Objects can be created as a result to a call to [MGHD](#), [MSGHD](#), [cMSGHD](#), [MCGHD](#), and [MGHFA](#).

### Slots

**index** Value of the index used for model selection (AIC or ICL or BIC or AIC3) for each G,the index used is specified by the user, if not specified AIC is used.

**BIC** Bayesian information criterion value.

**ICL** ICL index.

**AIC** AIC index.

**AIC3** AIC3 index.

**gpar** A list of the model parameters (in the rotated space for MCGHD).

**loglik** The log-likelihood values.

**map** A vector of integers indicating the maximum a posteriori classifications for the best model.

**par** Only for MCGHD. A list of the model parameters.

**z** A matrix giving the raw values upon which map is based.

### Methods

**plot** signature(x = "MixGHD") Provides plots of [MixGHD-class](#) by plotting the following elements:

- the value of the log likelihood for each iteration.
- Scatterplot of the data of all the possible couples of coordinates coloured according to the cluster. Only for less than 10 variables.
- If the number of variables is two: scatterplot and contour plot of the data coloured according to the cluster

**summary** `summary(x = "MixGHD")`.

Provides a summary of [MixGHD-class](#) objects by printing the following elements:

- The number components used for the model
- BIC;
- AIC;
- AIC3;
- ICL;
- A table with the number of element in each cluster.

### Author(s)

Cristina Tortora, Aisha ElSherbiny, Ryan P. Browne, Brian C. Franczak, and Paul D. McNicholas.  
Maintainer: Cristina Tortora <crisrina.tortora@sjsu.edu>

### See Also

[MixGHD-class](#)

### Examples

```
##loading bankruptcy data
data(bankruptcy)

##model estimation
#res=MCGHD(data=bankruptcy[,2:3],G=2,method="kmedoids",max.iter=30)
#result
#plot(res)
#summary(res)
```

---

MixGHD-class

*Class MixGHD.*

---

### Description

This class pertains to results of the application of function [MGHD](#),[MCGHD](#),[MSGHD](#),[cMSGHD](#).

### Details

Plot the loglikelihood vale for each iteration of the EM algorithm. If  $p=2$  it shows a contour plot. If  $2 < p < 10$  shows a splom of the data colored according to the cluster membership.

**Slots**

**Index** Bayesian information criterion value for each combination of G and q.

**BIC** Bayesian information criterion value.

**gpar** A list of the model parameters.

**loglik** The log-likelihood values.

**map** A vector of integers indicating the maximum a posteriori classifications for the best model.

**z** A matrix giving the raw values upon which map is based.

**method** A string indicating the used method: MGH, MGHFA, MSGHD, cMSGHD, MCGHD.

**data** A matrix or data frame such that rows correspond to observations and columns correspond to variables.

**par** (only for MCGHD) A list of the model parameters in the rotated space.

**Methods**

`signature(x = "MixGHD", y = "missing")` S4 method for plotting objects of [MixGHD-class](#).

**Author(s)**

Cristina Tortora, Aisha ElSherbiny, Ryan P. Browne, Brian C. Franczak, and Paul D. McNicholas.  
Maintainer: Cristina Tortora <crisrina.tortora@sjsu.edu>

**See Also**

[MixGHD-class](#), [MGH](#), [MCGHD](#), [MSGHD](#), [cMSGHD](#), [MGHFA](#)

**Examples**

```
##loading banknote data
data(bankruptcy)

##model estimation
model=MSGHD(bankruptcy[,2:3],G=2,max.iter=30)

#result
summary(model)
plot(model)
```

---

MSGHD	<i>Mixture of multiple scaled generalized hyperbolic distributions (MSGHD).</i>
-------	---

---

### Description

Carries out model-based clustering using the mixture of multiple scaled generalized hyperbolic distributions.

### Usage

```
MSGHD(data=NULL, gpar0=NULL, G=2, max.iter=100, label=NULL, eps=1e-2,
method="km", scale=TRUE, nr=10, modelSel="AIC")
```

### Arguments

data	A n x p matrix or data frame such that rows correspond to observations and columns correspond to variables.
gpar0	(optional) A list containing the initial parameters of the mixture model. See the 'Details' section.
G	The range of values for the number of clusters.
max.iter	(optional) A numerical parameter giving the maximum number of iterations each EM algorithm is allowed to use.
label	( optional) A n dimensional vector, if label[i]=k then observation i belongs to group k, If label[i]=0 then observation i has no known group, if NULL then the data has no known groups.
eps	(optional) A number specifying the epsilon value for the convergence criteria used in the EM algorithms. For each algorithm, the criterion is based on the difference between the log-likelihood at an iteration and an asymptotic estimate of the log-likelihood at that iteration. This asymptotic estimate is based on the Aitken acceleration.
method	( optional) A string indicating the initialization criterion, if not specified kmeans clustering is used. Alternative methods are: hierarchical "hierarchical", random "random", and model based "modelBased" clustering
scale	( optional) A logical value indicating whether or not the data should be scaled, true by default.
nr	( optional) A number indicating the number of starting value when random is used, 10 by default.
modelSel	( optional) A string indicating the model selection criterion, if not specified AIC is used. Alternative methods are: BIC, ICL, and AIC3

### Details

The arguments gpar0, if specified, is a list structure containing at least one p dimensional vector mu, alpha and phi, a p x p matrix gamma, and a p x 2 matrix cpl containing the vector omega and the vector lambda.

**Value**

A S4 object of class `MixGHD` with slots:

<code>index</code>	Value of the index used for model selection (AIC or ICL or BIC or AIC3) for each G, the index used is specified by the user, if not specified AIC is used.
<code>BIC</code>	Bayesian information criterion.
<code>ICL</code>	Integrated completed likelihood.
<code>AIC</code>	Akaike information criterion.
<code>AIC3</code>	Akaike information criterion 3.
<code>gpar</code>	A list of the model parameters
<code>loglik</code>	The log-likelihood values.
<code>map</code>	A vector of integers indicating the maximum a posteriori classifications for the best model.
<code>z</code>	A matrix giving the raw values upon which map is based.

**Author(s)**

Cristina Tortora, Aisha ElSherbiny, Ryan P. Browne, Brian C. Franczak, and Paul D. McNicholas.  
 Maintainer: Cristina Tortora <crisrina.tortora@sjsu.edu>

**References**

C. Tortora, B.C. Franczak, R.P. Browne, and P.D. McNicholas (2019). A Mixture of Coalesced Generalized Hyperbolic Distributions. *Journal of Classification* 36(1) 26-57. C. Tortora, R. P. Browne, A. ElSherbiny, B. C. Franczak, and P. D. McNicholas (2021). Model-Based Clustering, Classification, and Discriminant Analysis using the Generalized Hyperbolic Distribution: MixGHD R package, *Journal of Statistical Software* 98(3) 1–24, <doi:10.18637/jss.v098.i03>.

**See Also**

[MGHD](#)

**Examples**

```
##loading banknote data
data(banknote)

##model estimation
model=MSGHD(banknote[,2:7],G=2,max.iter=30)

#result
table(banknote[,1],model@map)
summary(model)
plot(model)
```

---

plot

*Plot objects of class MixGHD.*

---

### Description

Plots the loglikelihood function and for  $p < 10$  shows the splom of the data.

### Usage

```
## S4 method for signature 'MixGHD'  
plot(x,y)
```

### Arguments

x                    A object of [MixGHD-class](#);  
y                    Not used; for compatibility with generic plot.

### Details

Plot the loglikelihood vale for each iteration of the EM algorithm. If  $p=2$  it shows a contour plot. If  $2 < p < 10$  shows a splom of the data colored according to the cluster membership.

### Methods

`signature(x = "MixGHD", y = "missing")` S4 method for plotting objects of [MixGHD-class](#).

### Author(s)

Cristina Tortora. Maintainer: Cristina Tortora <[cristina.tortora@sjsu.edu](mailto:cristina.tortora@sjsu.edu)>

### See Also

[MixGHD-class](#), [MGHD](#), [MCGHD](#), [MSGHD](#), [cMSGHD](#), [MGHFA](#)

### Examples

```
##loading banknote data  
data(bankruptcy)  
  
##model estimation  
model=MCGHD(bankruptcy[,2:3],G=2,max.iter=30)  
  
#result  
  
plot(model)
```

---

predict	<i>Membership prediction for objects of class MixGHD</i>
---------	--

---

**Description**

Cluster membership

**Usage**

```
## S4 method for signature 'MixGHD'
predict(object)
```

**Arguments**

object            An S4 object of class [MixGHD](#).

**Value**

The cluster membership

**Author(s)**

Cristina Tortora Maintainer: Cristina Tortora <crisrina.tortora@sjsu.edu>

**Examples**

```
##loading bankruptcy data
data(bankruptcy)

##model estimation
res=MCGHD(data=bankruptcy[,2:3],G=2,method="kmedoids",max.iter=30)
#rcoefficients of the model
predict(res)
```

---

rCGHD	<i>Pseudo random number generation from a coalesced generalized hyperbolic distribution (MSGHD).</i>
-------	--

---

**Description**

Generate n pseudo random numbers from a p dimensional coalesced generalized hyperbolic distribution.

**Usage**

```
rCGHD(n,p,mu=rep(0,p),alpha=rep(0,p),sigma=diag(p),omega=1,lambda=0.5
,omegav=rep(1,p),lambdav=rep(0.5,p),wg=0.5)
```

**Arguments**

n	number of observations.
p	number of variables.
mu	(optional) the p dimensional mean
alpha	(optional) the p dimensional skewness parameter alpha
sigma	(optional) the p x p dimensional scale matrix
lambda	(optional) the 1 dimensional index parameter lambda
omega	(optional) the 1 dimensional concentration parameter omega
omegav	(optional) the p dimensional concentration parameter omega
lambdav	(optional) the p dimensional index parameter lambda
wg	(optional) the weight

**Details**

The default values are: 0 for the mean and the skewness parameter alpha,  $\text{diag}(p)$  for sigma, 1 for omega, and 0.5 for lambda.

**Value**

A n times p matrix of numbers pseudo randomly generated from a coalesced generalized hyperbolic distribution

**Author(s)**

Cristina Tortora, Aisha ElSherbiny, Ryan P. Browne, Brian C. Franczak, and Paul D. McNicholas.  
Maintainer: Cristina Tortora <cristina.tortora@sjsu.edu>

**References**

C. Tortora, B.C. Franczak, R.P. Browne, and P.D. McNicholas (2019). A Mixture of Coalesced Generalized Hyperbolic Distributions. *Journal of Classification* (to appear).

**Examples**

```
data=rCGHD(300,2,alpha=c(2,-2),omegav=c(2,2),omega=3)
plot(data)
```

---

rGHD	<i>Pseudo random number generation from a generalized hyperbolic distribution (GHD).</i>
------	--

---

### Description

Generate n pseudo random numbers from a p dimensional generalized hyperbolic distribution.

### Usage

```
rGHD(n,p, mu=rep(0,p),alpha=rep(0,p),sigma=diag(p),omega=1,lambda=0.5)
```

### Arguments

n	number of observations.
p	number of variables.
mu	(optional) the p dimensional mean
alpha	(optional) the p dimensional skewness parameter alpha
sigma	(optional) the p x p dimensional scale matrix
omega	(optional) the unidimensional concentration parameter omega
lambda	(optional) the unidimensional index parameter lambda

### Details

The default values are: 0 for the mean and the skewness parameter alpha, diag(p) for sigma, 1 for omega, and 0.5 for lambda.

### Value

A n times p matrix of numbers pseudo randomly generated from a generalized hyperbolic distribution

### Author(s)

Cristina Tortora, Aisha ElSherbiny, Ryan P. Browne, Brian C. Franczak, and Paul D. McNicholas.  
 Maintainer: Cristina Tortora <cristina.tortora@sjsu.edu>

### References

R.P. Browne, and P.D. McNicholas (2015). A Mixture of Generalized Hyperbolic Distributions. *Canadian Journal of Statistics*, 43.2 176-198

### Examples

```
data=rGHD(300,2,alpha=c(2,-2))
plot(data)
```

---

rMSGHD	<i>Pseudo random number generation from a multiple-scaled generalized hyperbolic distribution (MSGHD).</i>
--------	--

---

**Description**

Generate  $n$  pseudo random numbers from a  $p$  dimensional multiple-scaled generalized hyperbolic distribution.

**Usage**

```
rMSGHD(n,p, mu=rep(0,p), alpha=rep(0,p), sigma=diag(p), omegav=rep(1,p), lambdav=rep(0.5,p))
```

**Arguments**

<code>n</code>	number of observations.
<code>p</code>	number of variables.
<code>mu</code>	(optional) the $p$ dimensional mean
<code>alpha</code>	(optional) the $p$ dimensional skewness parameter $\alpha$
<code>sigma</code>	(optional) the $p \times p$ dimensional scale matrix
<code>omegav</code>	(optional) the $p$ dimensional concentration parameter $\omega$
<code>lambdav</code>	(optional) the $p$ dimensional index parameter $\lambda$

**Details**

The default values are: 0 for the mean and the skewness parameter  $\alpha$ ,  $\text{diag}(p)$  for  $\sigma$ , 1 for  $\omega$ , and 0.5 for  $\lambda$ .

**Value**

A  $n$  times  $p$  matrix of numbers pseudo randomly generated from a generalized hyperbolic distribution

**Author(s)**

Cristina Tortora, Aisha ElSherbiny, Ryan P. Browne, Brian C. Franczak, and Paul D. McNicholas.  
Maintainer: Cristina Tortora <cristina.tortora@sjsu.edu>

**References**

C. Tortora, B.C. Franczak, R.P. Browne, and P.D. McNicholas (2019). A Mixture of Coalesced Generalized Hyperbolic Distributions. *Journal of Classification* (to appear).

**Examples**

```
data=rMSGHD(300,2,alpha=c(2,-2),omegav=c(2,2))
plot(data)
```

---

sonar

*Sonar data*

---

### Description

The data report the patterns obtained by bouncing sonar signals at various angles and under various conditions. There are 208 patterns in all, 111 obtained by bouncing sonar signals off a metal cylinder and 97 obtained by bouncing signals off rocks. Each pattern is a set of 60 numbers (variables) taking values between 0 and 1.

### Usage

```
data(sonar)
```

### Format

A data frame with 208 observations and 61 columns. The first 60 columns contain the variables. The 61st column gives the material: 1 rock, 2 metal.

### Source

UCI machine learning repository

### References

R.P. Gorman and T. J. Sejnowski (1988) Analysis of hidden units in a layered network trained to classify sonar targets. *Neural Networks* **1**: 75-89

---

summary

*Plot objects of class MixGHD.*

---

### Description

Methods for function summary aimed at summarizing the S4 classes included in the [MixGHD](#)-package

### Arguments

object            A object of [MixGHD-class](#).

### Methods

signature(object = "MixGHD") S4 method for summarizing objects of [MixGHD-class](#).

### Author(s)

Cristina Tortora. Maintainer: Cristina Tortora <[cristina.tortora@sjsu.edu](mailto:cristina.tortora@sjsu.edu)>

*summary*

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### **See Also**

[MixGHD](#) [MixGHD-class](#), [MGHD](#), [MCGHD](#), [MSGHD](#), [cMSGHD](#), [MGHFA](#)

### **Examples**

```
##loading banknote data
data(bankruptcy)

##model estimation
model=MSGHD(bankruptcy[,2:3],G=2,max.iter=30)

#result

summary(model)
```

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